Assessment of Grid-Induced Errors in HiLiftPW-1 Predictions Using Error Transport Equations

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Presentation Overview

- Analysis Objectives
  - All CFD solutions have errors: how can we quantify them?
  - How well do error prediction methods works?
  - How can we interpret and act on this information?
- Error Quantification Methods
  - Error Transport Equation (ETE) Solver
  - Error Function Library
  - Generalized Richardson Extrapolation
- Results (Case 1 Only – Grid Convergence Study)
  - *FUN3D* Grid Sequence
  - *USM3D* Grid Sequence
  - Mesh Adaptation using Error Sources
- Conclusions and Lessons Learned

Errors in CFD Analysis

- Solution errors inherent to any CFD analysis
  - Discretization errors (grid size/spacings, time step)
  - Modeling errors (turbulence, transition, *etc*.)
  - Usage, iterative convergence, coding errors, *etc*.
- Discretization errors generally most dominant and most readily controlled
- How do we check our results? Verification & Validation (V&V)
  - Compare with test data (validation)
  - Perform grid refinement study (verification)
  - Richardson extrapolation given 3 mesh sequence
- ***So what's the catch?***
  - We don't always have test data
  - Grid refinement studies are time-consuming and expensive (labor, CPU) for practical 3D problems
  - Richardson extrapolation isn't always applicable

Study Objectives

- Assess our current error quantification methodology for a real world application
  - Use Error Transport Equation (ETE) model to certify mesh-induced errors on grid sequences and as “single grid” error estimator
  - Use conventional Richardson extrapolation if and where possible as reference check
- Examine solutions from two established unstructured solvers
  - *FUN3D* solutions using grid sequence provided by committee
  - *USM3D* solutions and grid sequence provided by NASA LaRC
  - Spalart-Allmaras turbulence model used in both solution sets
  - Predict errors in surface pressure coefficient and integrated lift/drag coefficients
- Investigate effectiveness of mesh adaptation using error sources
- Identify shortcomings that remain to be addressed

What Are We Looking For?

- Goal is to establish solution verification method for any given grid and solution
- Reliable predictions of mesh-induced errors
  - Prediction of coarse-to-fine grid increments
  - Local and integrated quantities
- Error bars predicted by ETE solution and Error Functions should:
  1) Contain fine grid results
  2) Contain results of Richardson extrapolation
  3) Decrease in magnitude with grid refinement
  4) Not be overly conservative as to be unusable
- If fine grid results fall outside predicted error bars, it potentially indicates new flow features result from grid refinement
- If test data falls outside predicted solution and error bars, it potentially indicates a deficiency in physical modeling

Error Prediction / Quantification

- Error Transport Equation (ETE) Solver for steady state flows
  - Basic inviscid formulation follows Zhang *et al*[*]
  - Extensions to viscous, turbulent flows also developed[†]
  - k–ε, k–ω, Spalart-Allmaras models supported
  - Ideal gas and reacting, multi-component mixtures
- Error Function Library
  - Propagates predicted errors into derived variables of interest
  - PLOT3D functions, integrated functions, *etc*.
- Richardson Extrapolation utility
  - Generalized extrapolation approach for unstructured meshes
  - Useful for extrapolating 1D profiles as well as pointwise values
  - Reports order of convergence and grid convergence index (GCI)

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* Zhang, X.D., Trepanier, J.-Y., and Camarero, R., "A Posteriori Error Estimation for Finite-Volume Solutions of Hyperbolic Conservation Laws," *Computer Methods in Applied Mechanics and Engineering*, Vol. 185, 2000, pp. 1-19.

† Cavallo, P.A., Sinha, N., and O'Gara, M.R., "Viscous Error Transport Equation for Error Quantification of Turbulent Flows," AIAA Paper 2008-3851, 38th Fluid Dynamics Conference, Seattle, WA, June 23-26, 2008.

Inviscid ETE Formulation

- Postulate the finite volume solution on grid *h* does not solve PDE exactly, introduces a residual $\bar{R}$
- Subtract FV solution from PDE to obtain ETE

$$\frac{\partial}{\partial t}\iiint_{} \left(Q - Q^h\right) dV + \iint \left(\bar{F}\left(Q\right) - \bar{F}\left(Q^h\right)\right) \cdot \hat{n}\, dA = -\bar{R} \qquad \text{(Euler equations)}$$

$$\bar{F}\left(Q\right) - \bar{F}\left(Q^h\right) = \left[A\left(Q^h\right)\right]\left(Q - Q^h\right) = \left[A\left(Q^h\right)\right]\bar{\varepsilon} \qquad \text{(Roe)}$$

$$\frac{\partial}{\partial t}\iiint_{} \bar{\varepsilon} dV + \iint A\left(Q^h\right)\bar{\varepsilon}\, dA = -\bar{R}$$

- Where $\bar{\varepsilon} = Q - Q^h$ $\qquad \bar{R} = -\frac{1}{2}\iint \Delta\bar{F} \cdot \hat{n}\, dA$ (Following Zhang *et al.*)

- Error vector is difference between exact and FV solution
- ETE solved as post-processing step after CFD solution is obtained (CFD solution vector is frozen)

Viscous Extensions

- Inviscid ETE with difference in viscous fluxes

$$\frac{\partial}{\partial t}\iiint \bar{Q}-Q^h\ dV+\iint\ \bar{F}\ \bar{Q}\ -\bar{F}\ Q^h\ \ .\hat{n}\ dA=\iint\ \bar{G}\ \bar{Q}\ -\bar{G}\ Q^h\ \ .\hat{n}\ dA-\bar{R}_{INV}$$

- Consider we have RANS equations:
- Propose error in viscous flux is due to:
  - Error in $\bar{Q}$ assuming μ_t is correct
  - Error in μ_t assuming $\bar{Q}$ is correct
- So error in viscous flux is sum of:
  - Diffusion of error
  - Source/production term due to error in μ_t
- Final viscous form of ETE is

$$\tau_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\rho\overline{u_i'u_j'}=\ \mu+\mu_t\ \left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

$$\varepsilon_{-\rho\overline{u_i'u_j'}}=\mu_t\left(\frac{\partial \varepsilon_{u_i}}{\partial x_j}+\frac{\partial \varepsilon_{u_j}}{\partial x_i}\right)+\varepsilon_{\mu_t}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

$$\bar{G}\ Q\ -\bar{G}\ Q^h\ \ \hat{n}=\bar{G}\ \ \bar{\varepsilon}\ \ \hat{n}-\bar{R}_{TURB}$$

$$\boxed{\frac{\partial}{\partial t}\iiint \bar{\varepsilon}dV+\iint\ A\ Q^h\ \ \bar{\varepsilon}\ .\hat{n}\ dA=\iint \bar{G}\ \ \bar{\varepsilon}\ .\hat{n}\ dA-\bar{R}_{INV}-\bar{R}_{TURB}}$$

- Expanded to real gas multi-component mixtures as well

Error Function Library

- Define variable of interest in terms of Q or other previously computed variables

$$R = R\ x_1, x_2, \cdots, x_n$$

- Error propagation using simple summation of contributions from each independent variable

$$\varepsilon_R = \pm \sum \left(\frac{\partial R}{\partial x_i} \varepsilon_{x_i} \right)$$

- 64 functions derived and coded
  - Various PLOT3D functions
  - Integrated functions (flow rates, forces/moments)
  - Chemistry functions and species mass/mole fractions
- For each variable selected, we also produce:
  - Histogram of error distribution among the vertices
  - Global error measurement as volume-weighted average (suggested by Baker*)

$$\overline{\varepsilon}_g = \sum_{i=1}^{Nverts} \varepsilon_i V_i \Big/ \sum_{i=1}^{Nverts} V_i$$

➢ ***Error Function Library vastly increases utility of ETE results***

* Baker, T.J., "Mesh Adaptation Strategies for Problems in Fluid Dynamics," *Finite Elements in Analysis and Design*, Vol. 25, No. 3-4, 1997, pp. 243-273.

Available Error Functions

Density	Pressure		Temperature	Enthalpy, Entropy	Energy	Chemistry
rho	P	pitot	T	h	e	gamma
norm(rho)	norm(P)	pitotratio	norm(T)	norm(h)	norm(e)	rgas
log(norm(rho))	log(norm(P))	q	log(norm(T))	h0	e0	yjet
rho0	P0	Cp	T0	norm(h0)	norm(e0)	yext
norm(rho0)	norm(P0)	Cp0	norm(T0)	s	rhoe0	sp-NN
					ke	x-NN
					norm(ke)	y-NN
Momentum	**Velocity**		**Turbulence**	**Flow rate**	**Aerodynamics**	
rhou	u	a	k	mdot	forces	
rhov	v	M	epsilon	vdot	moments	
rhow	w	crossflow	mut		CLCD	
momentum*	vel*	vprime*	mul		tauwall*	
	mag(vel)		SA		Cf	

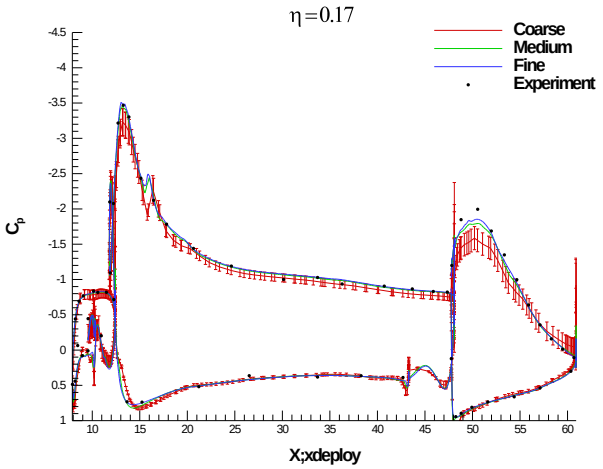
* Vector function

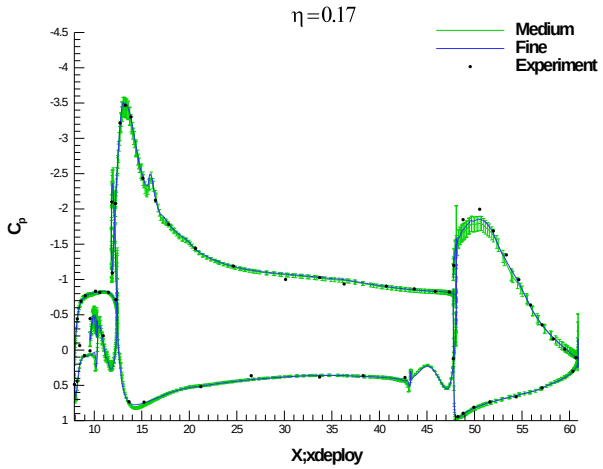
Richardson Extrapolation

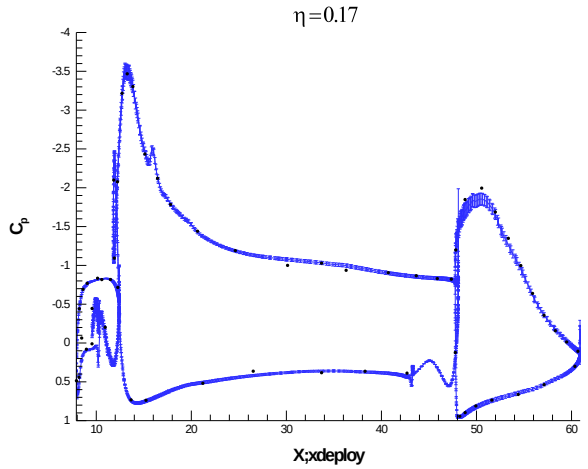
- Conventional approach for solution verification
- Generalized approach of Baker* utilized
  - Does not assume uniform refinement ratios
  - Mesh width defined by number of vertices
  - Solve for order of convergence *p*
- Utility reports GCI, extrapolated values, may be used for 1D profiles
- Restriction: solution ***must*** exhibit monotone behavior
- Potential issues with its applicability:
  - Monotonic behavior may be difficult to obtain
  - Mesh refinement may introduce new features
  - More than 3 meshes may be needed – rapidly loses practicality
  - Solution metrics may converge differently
  - Some metrics may not exhibit monotone behavior while others do

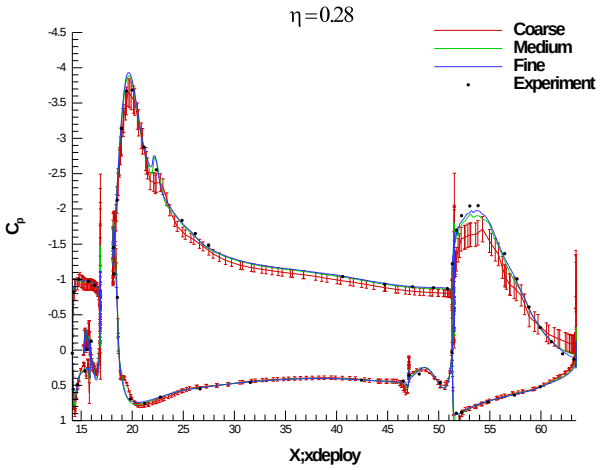
* Baker, T.J., "On the Relationship between Mesh Refinement and Solution Accuracy," AIAA Paper 2005-4875, 17th Computational Fluid Dynamics Conference, Toronto, Canada, June 6-9, 2005.

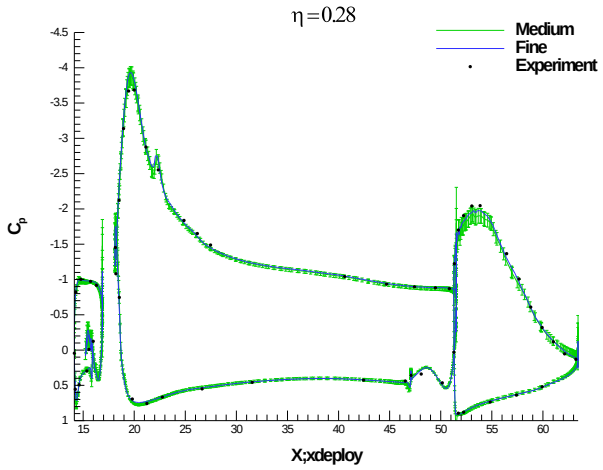
FUN3D Results, α=13° (1)

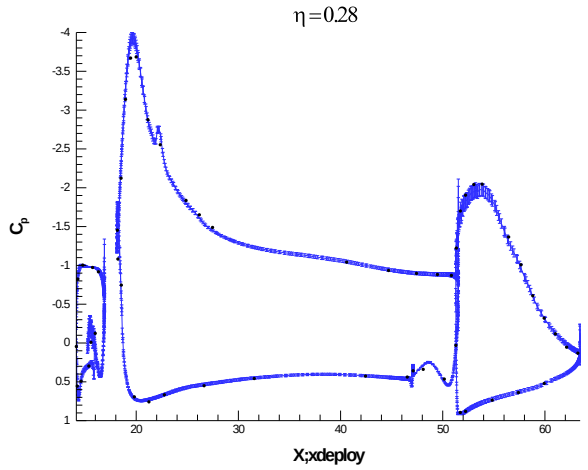












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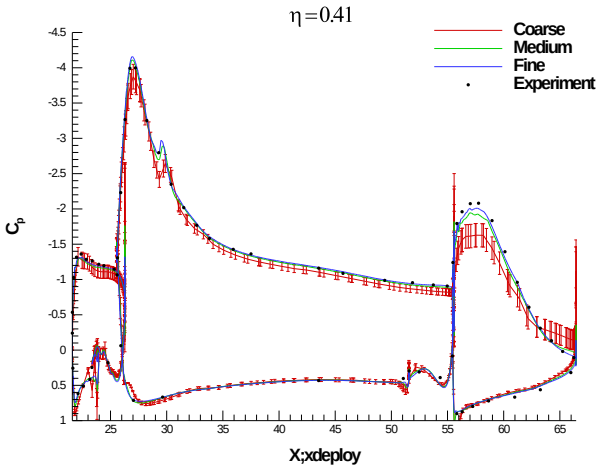

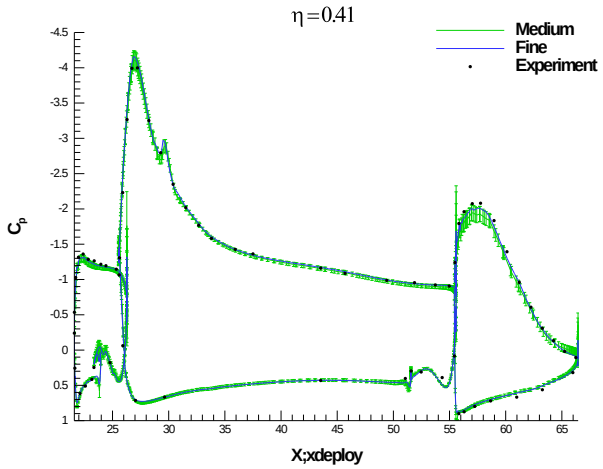

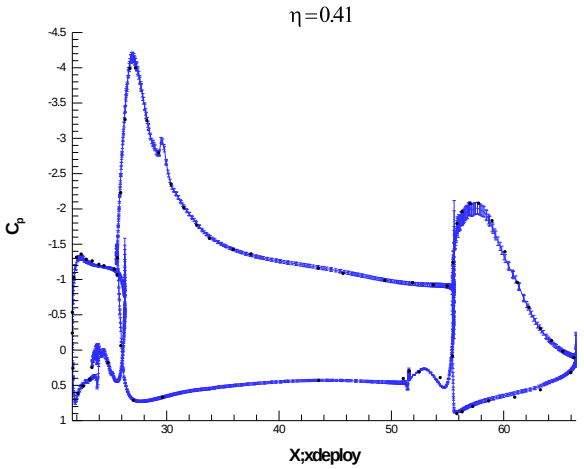

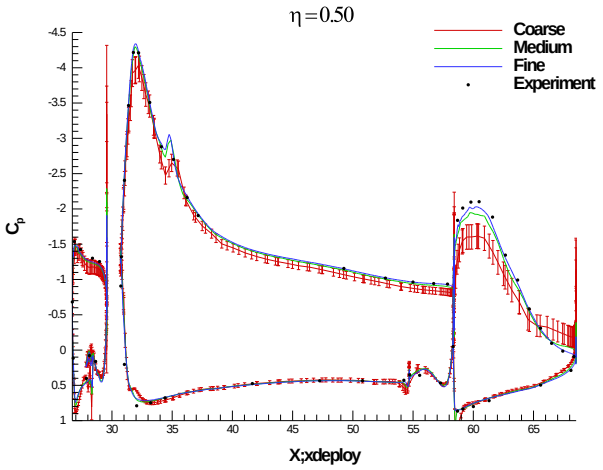

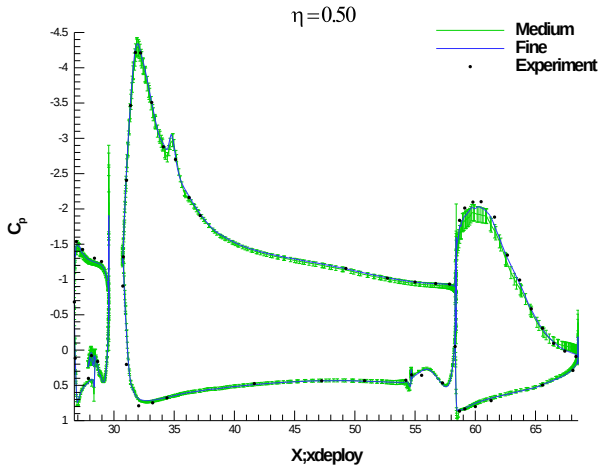

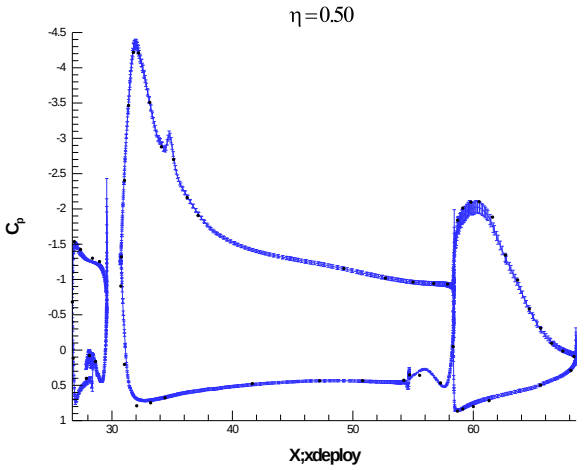

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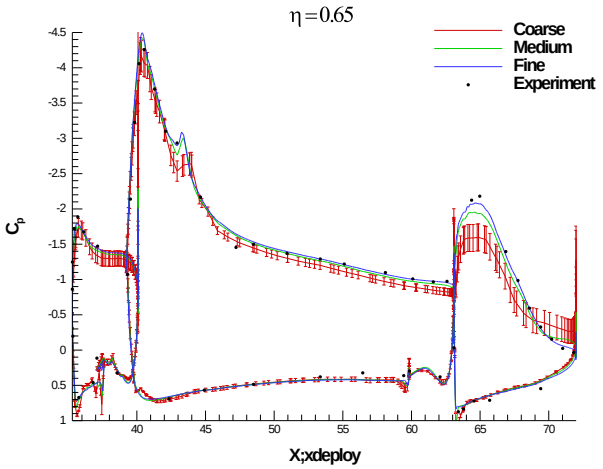

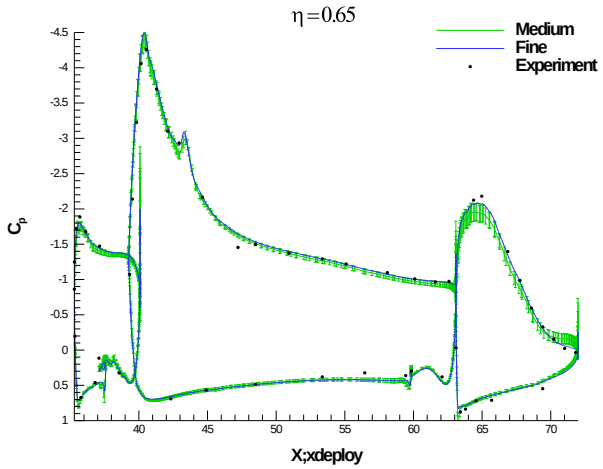

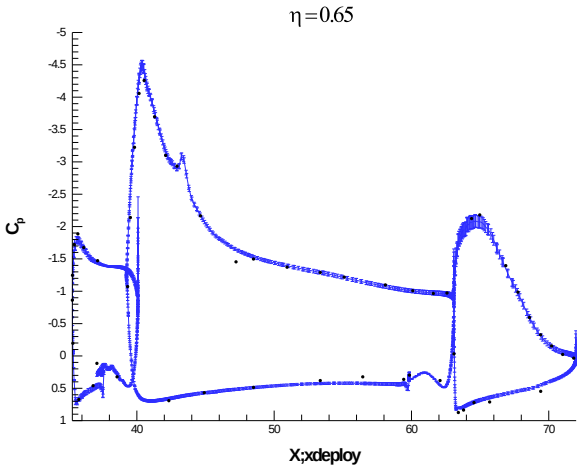

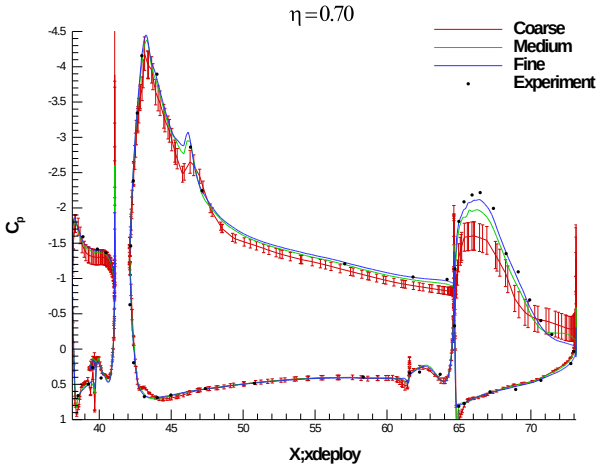

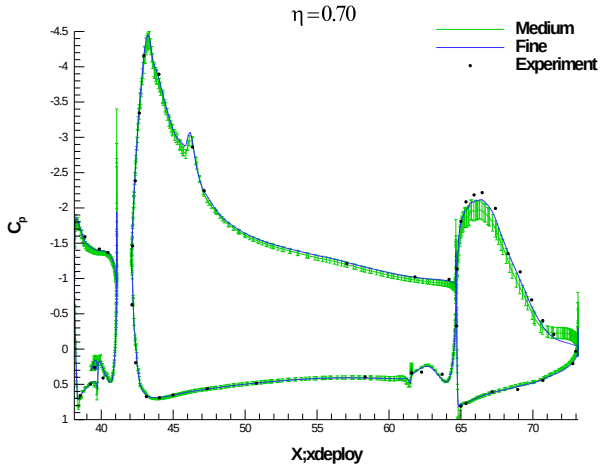

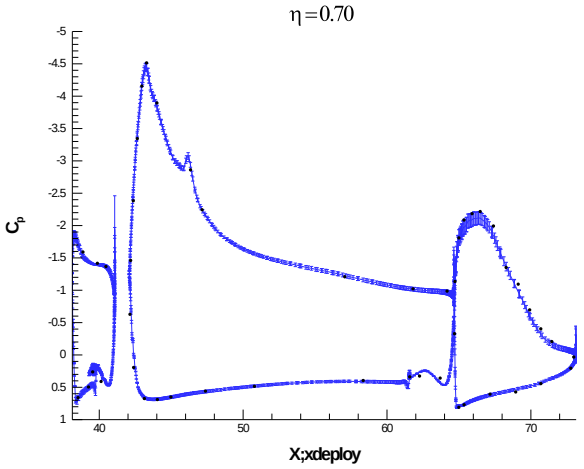

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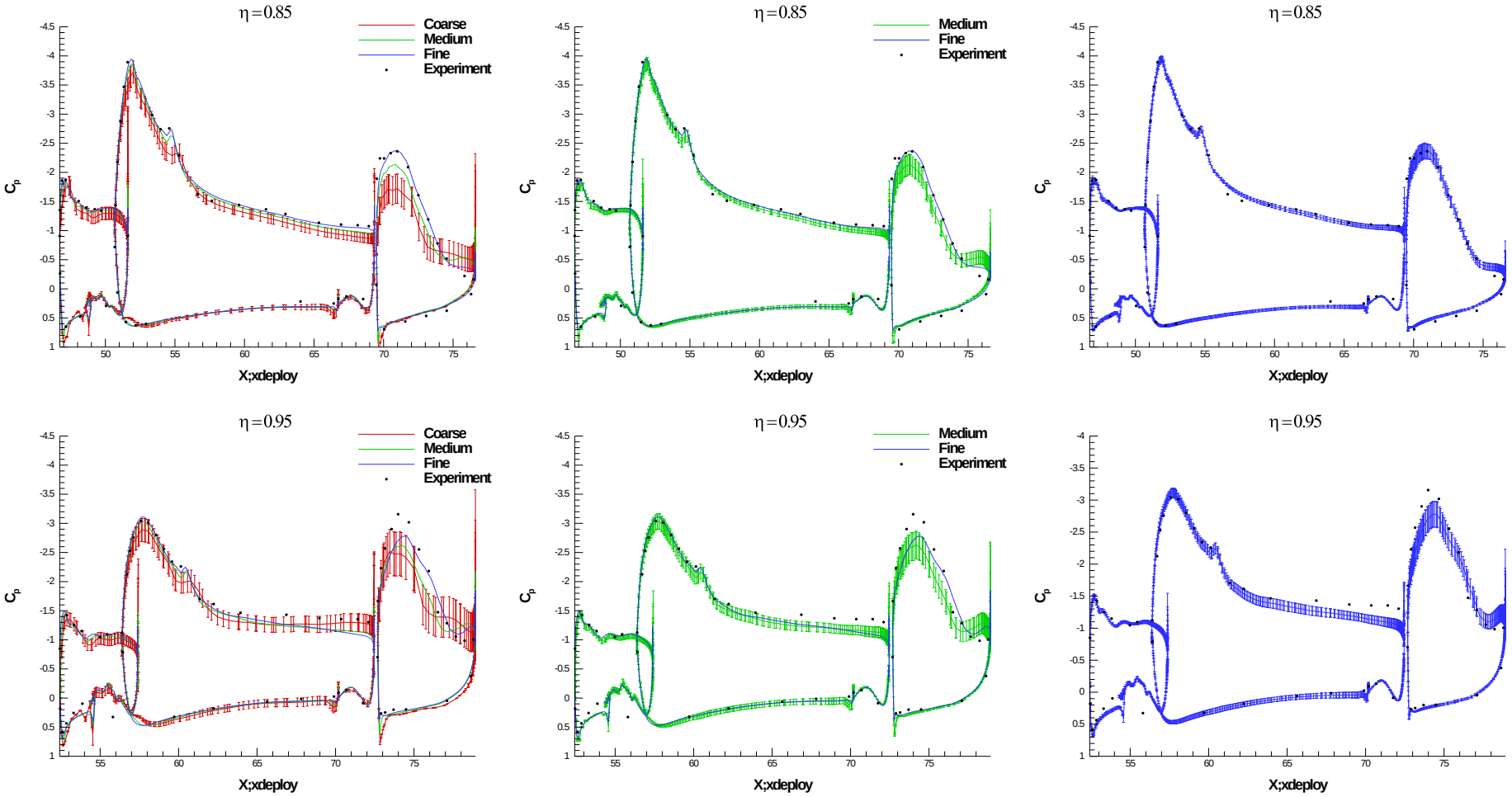
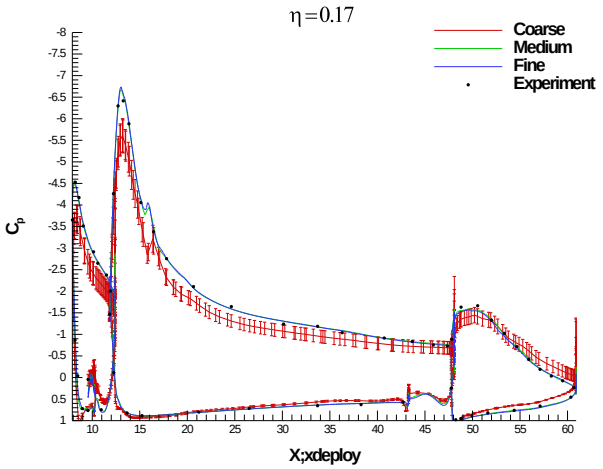
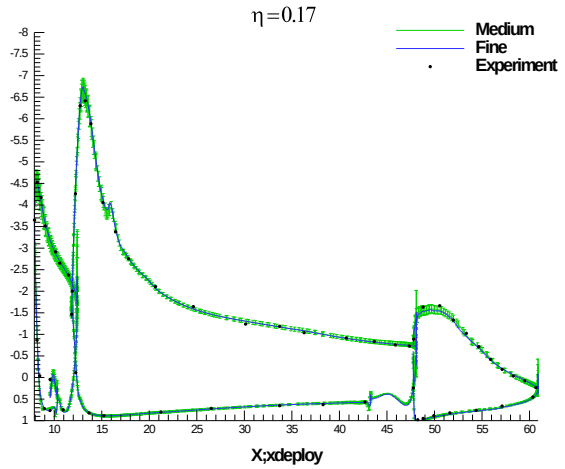
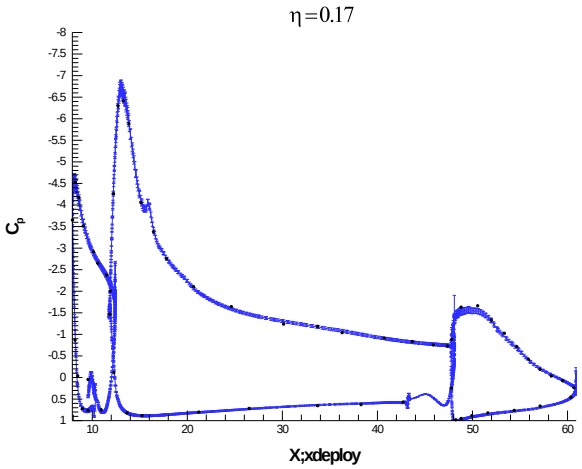
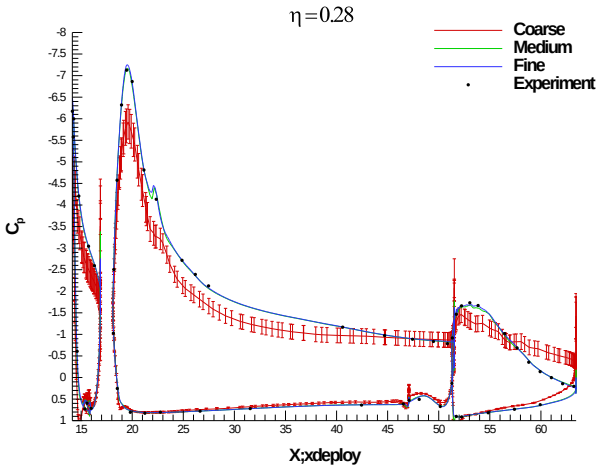


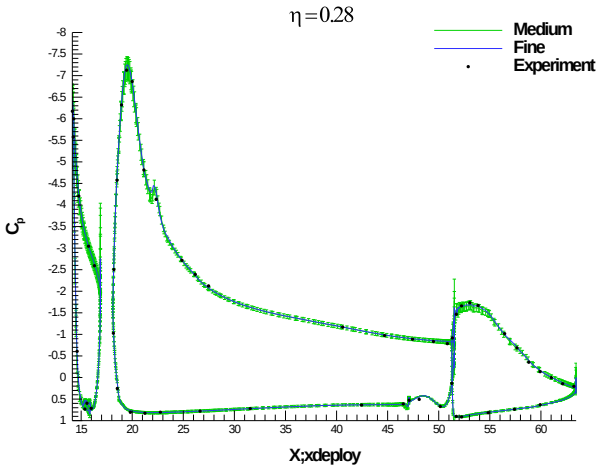
FUN3D Results, α=28° (1)

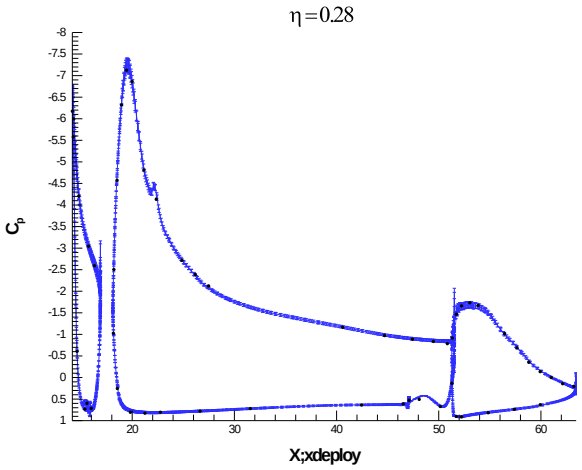












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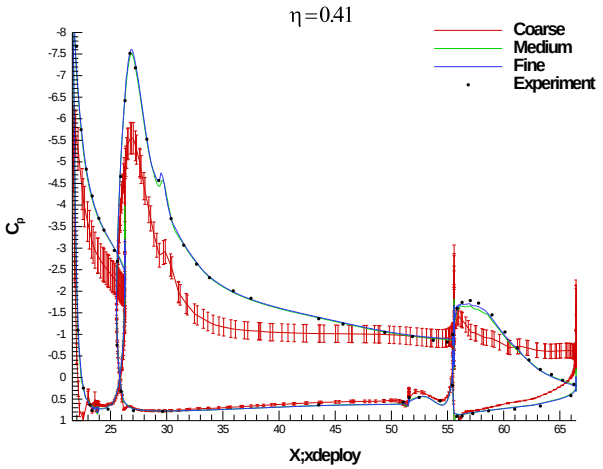

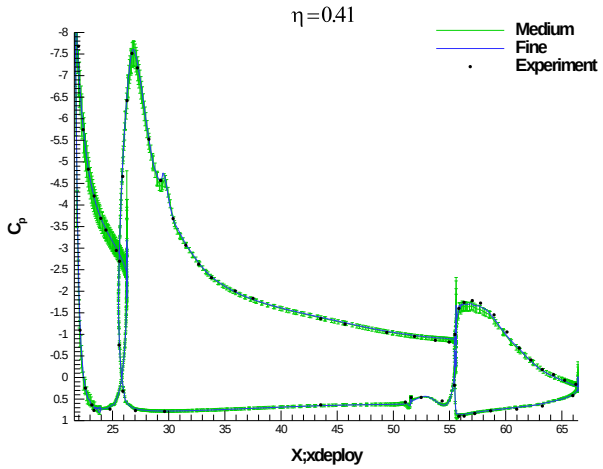

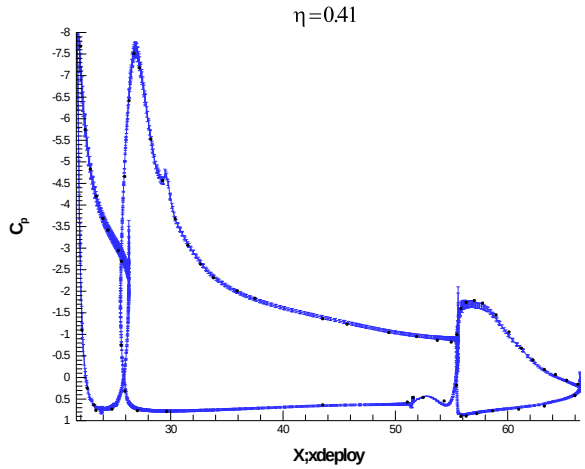

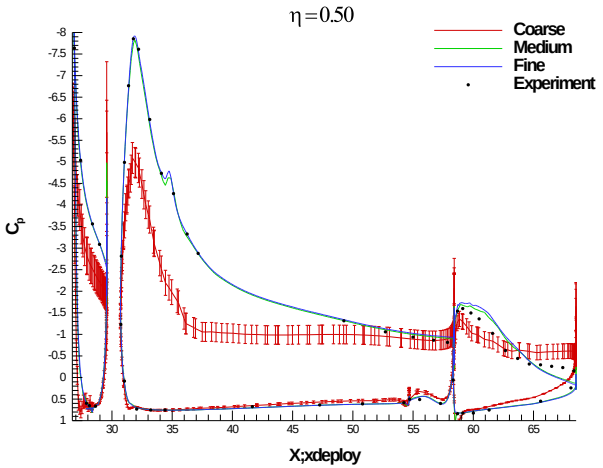

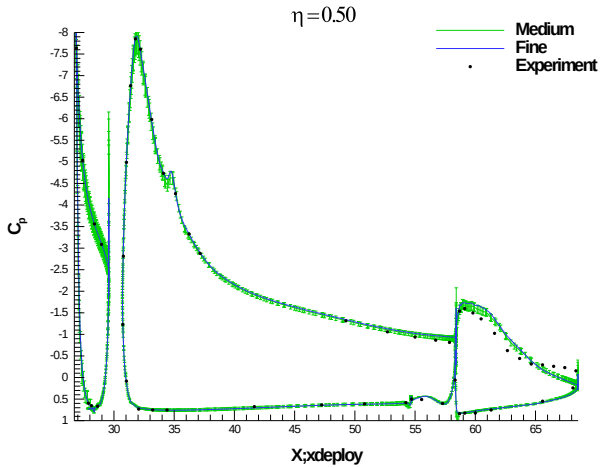

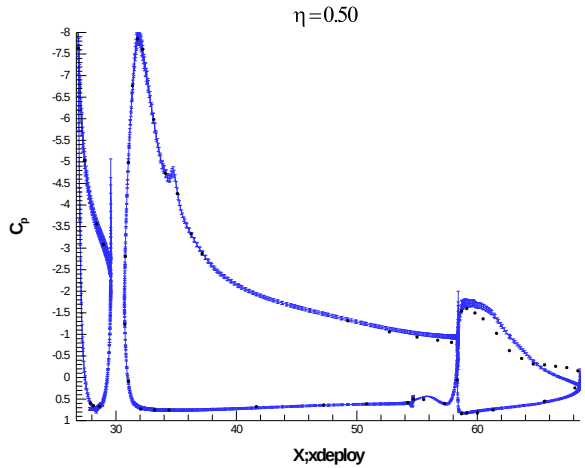

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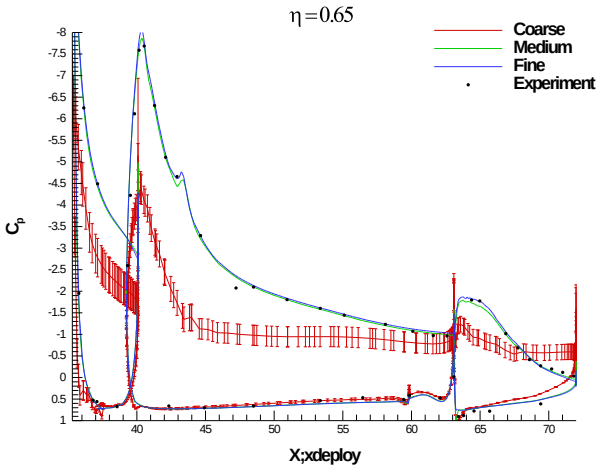
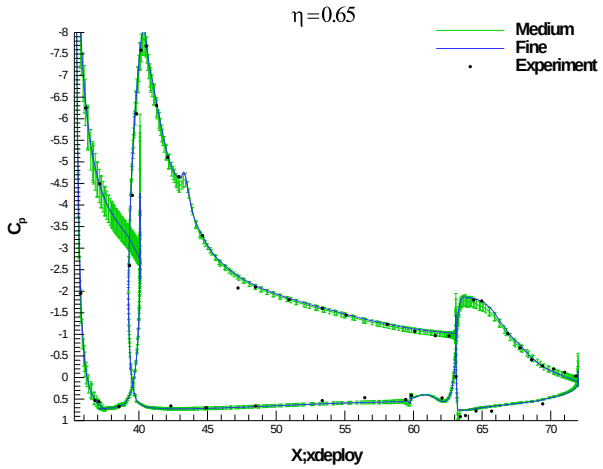
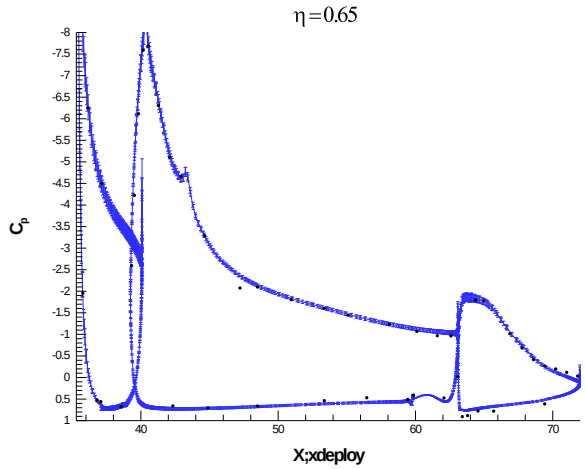
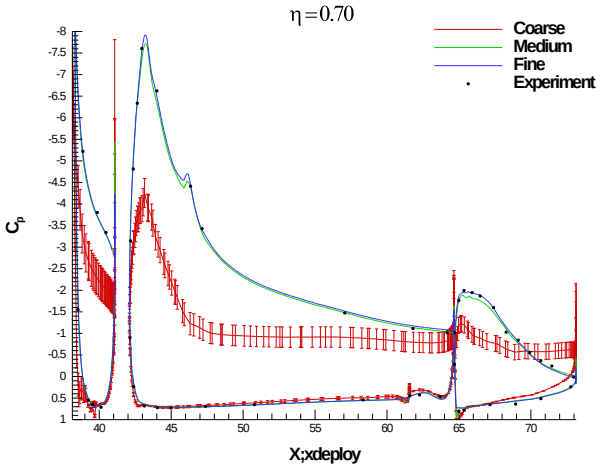
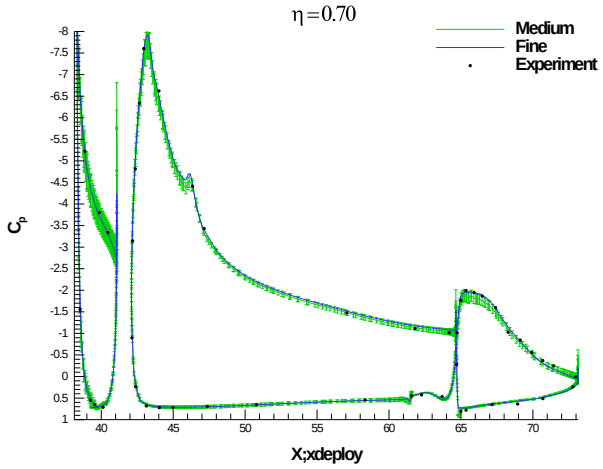
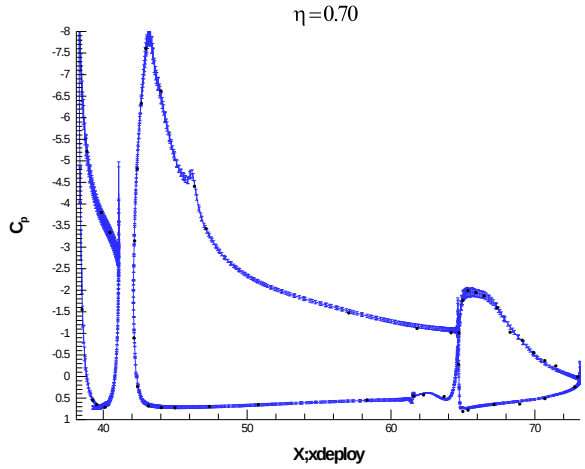




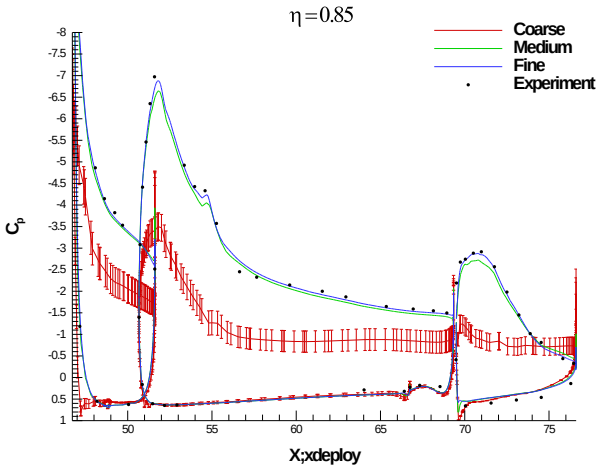


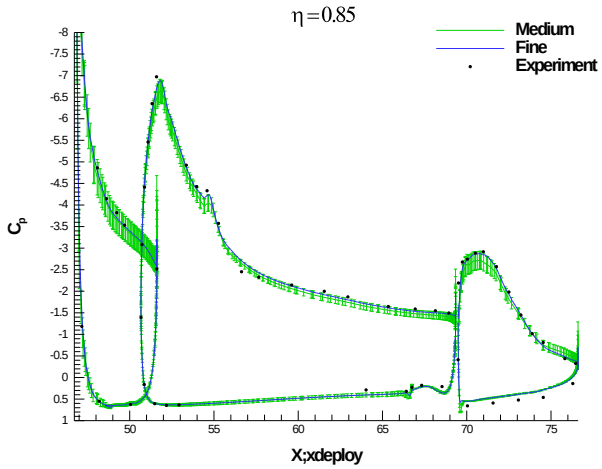


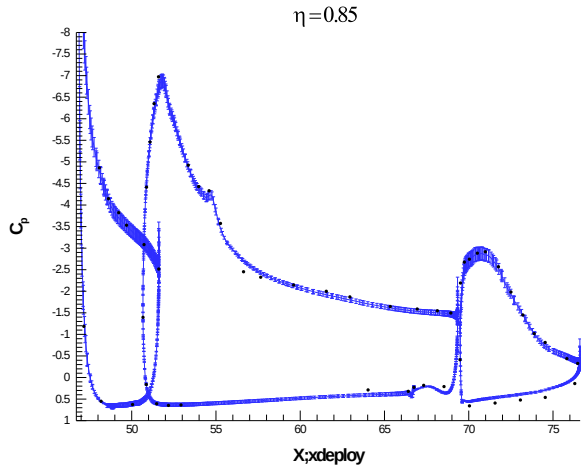


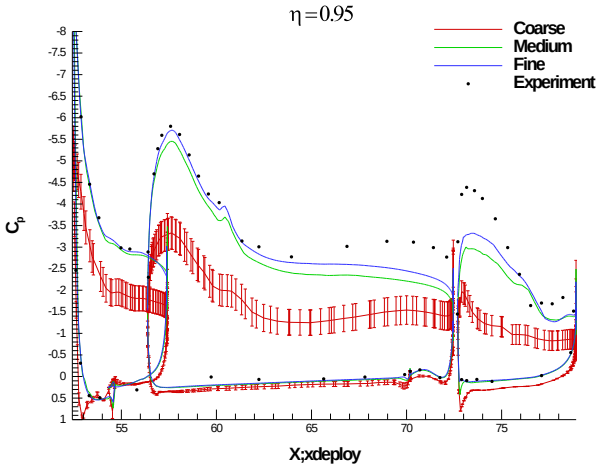


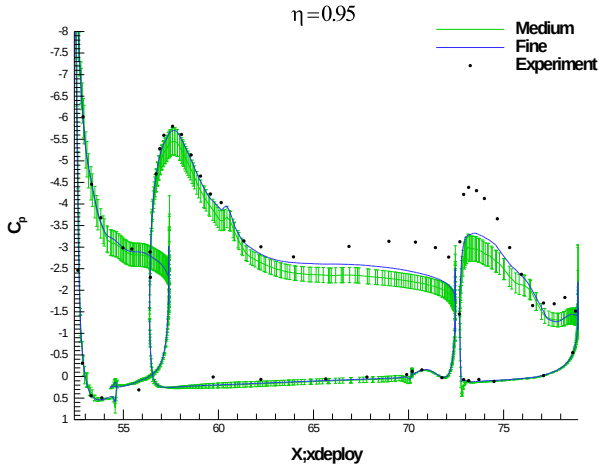
FUN3D Results, α=28° (4)

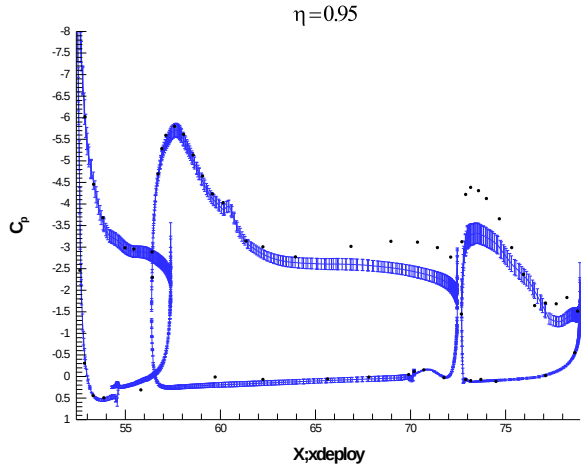






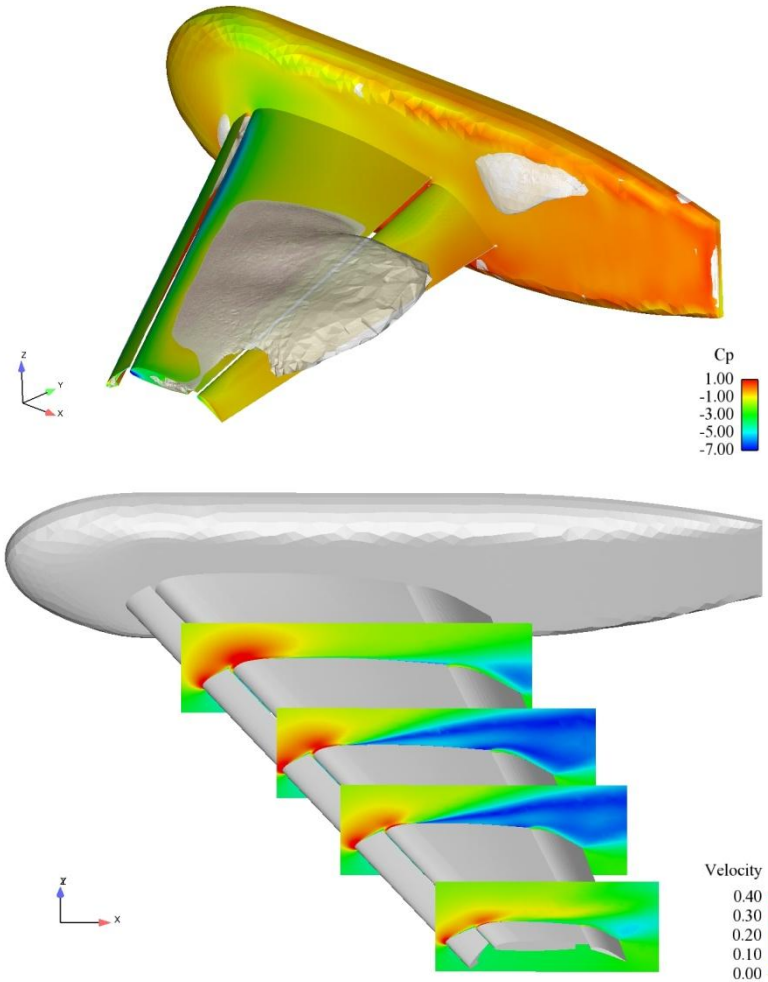




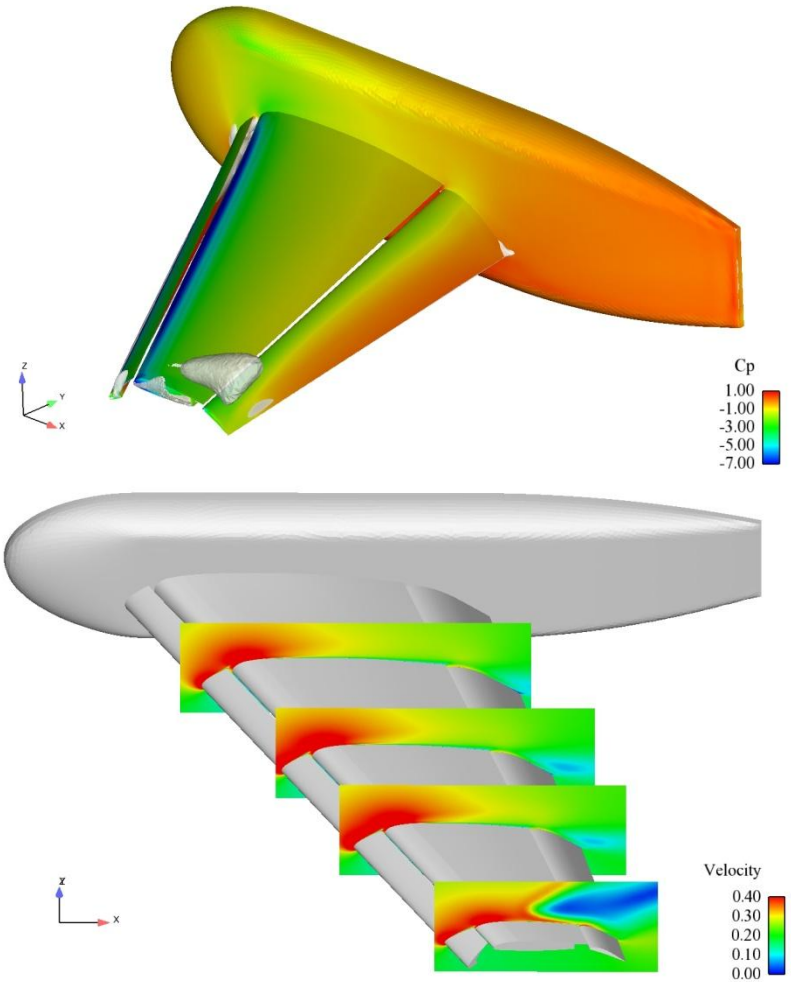


FUN3D Results, α=28° (5)

Coarse Grid

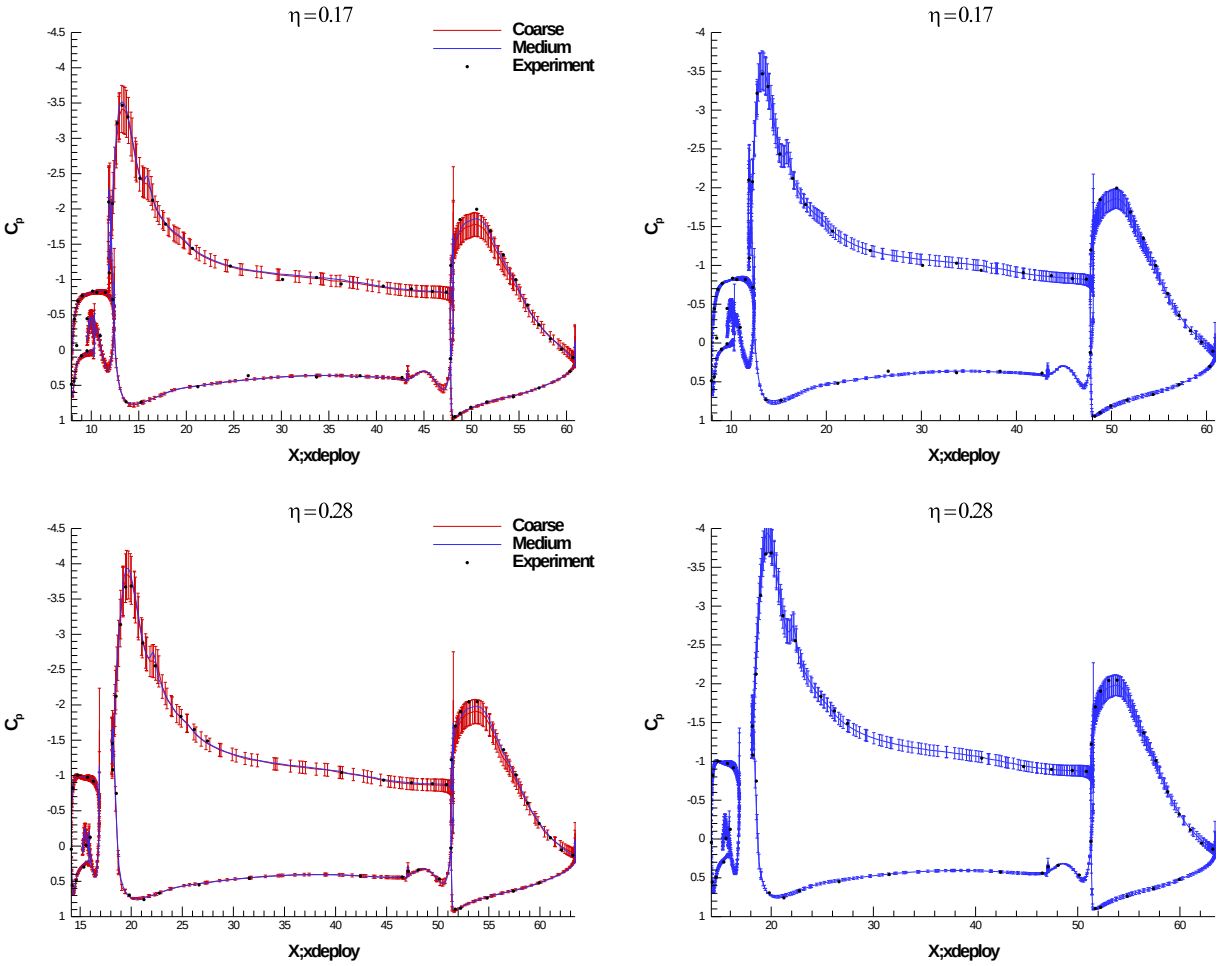


Medium Grid

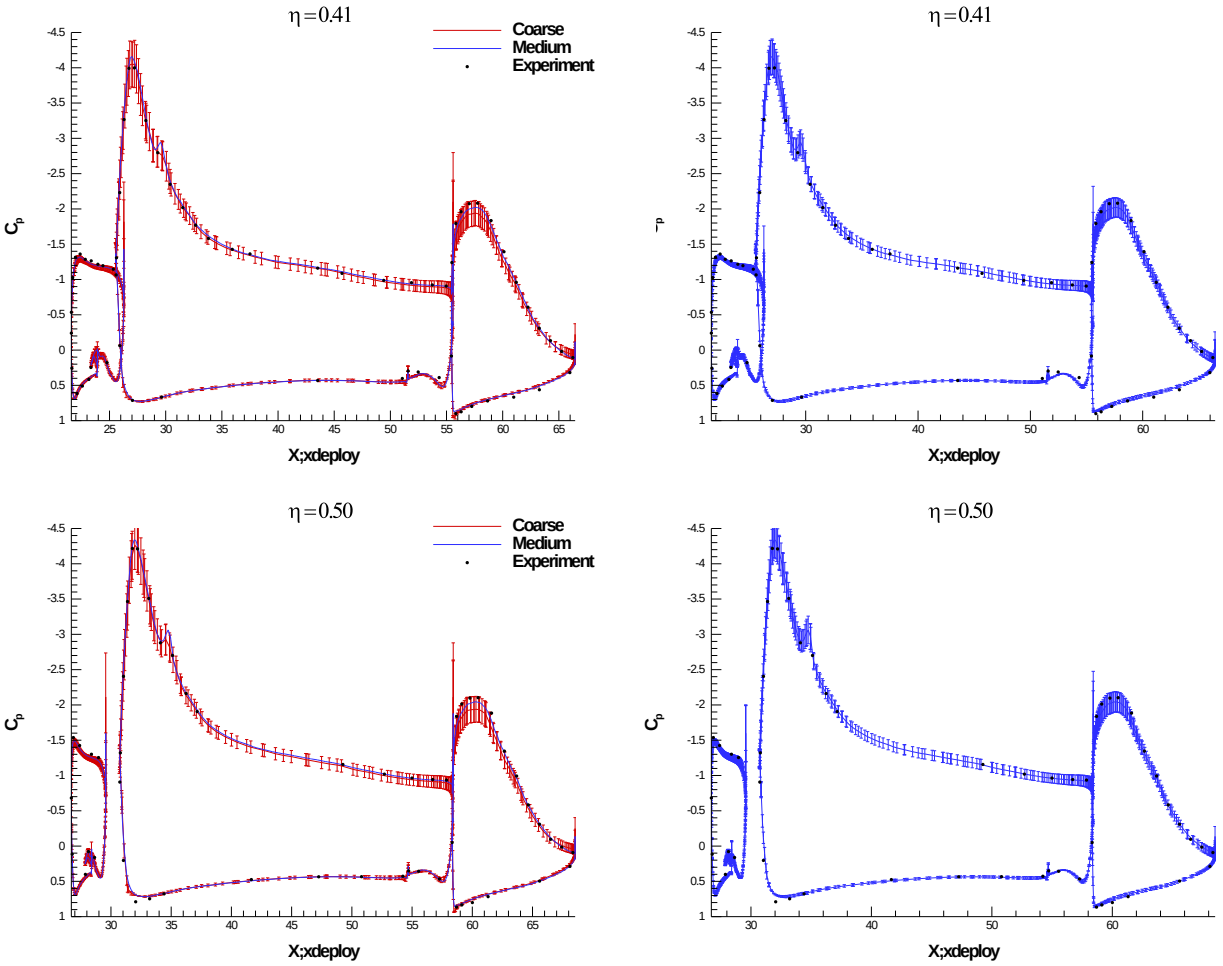


USM3D Results, α=13° (1)

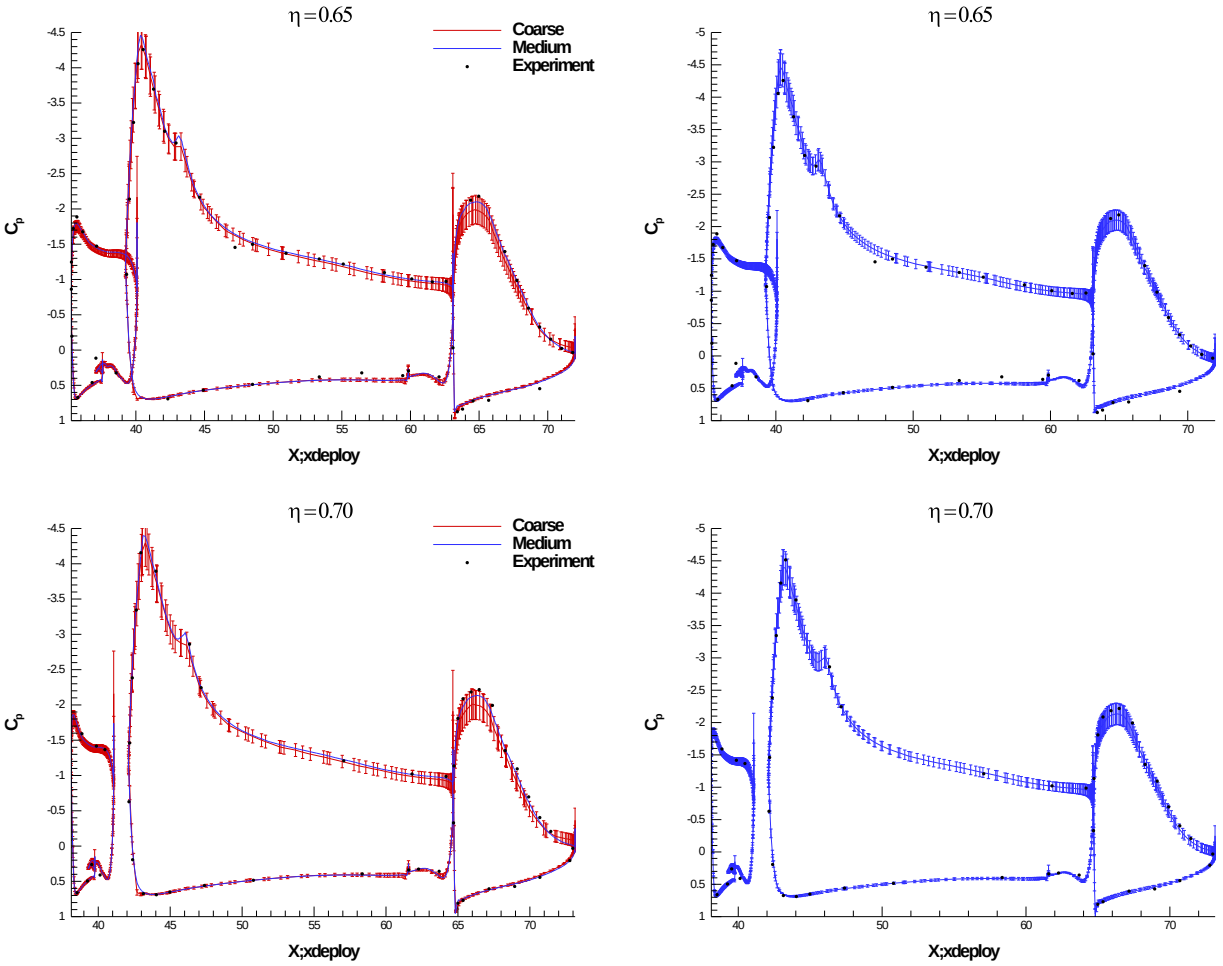
Note: unable to obtain fine grid result (difficulty processing grid)



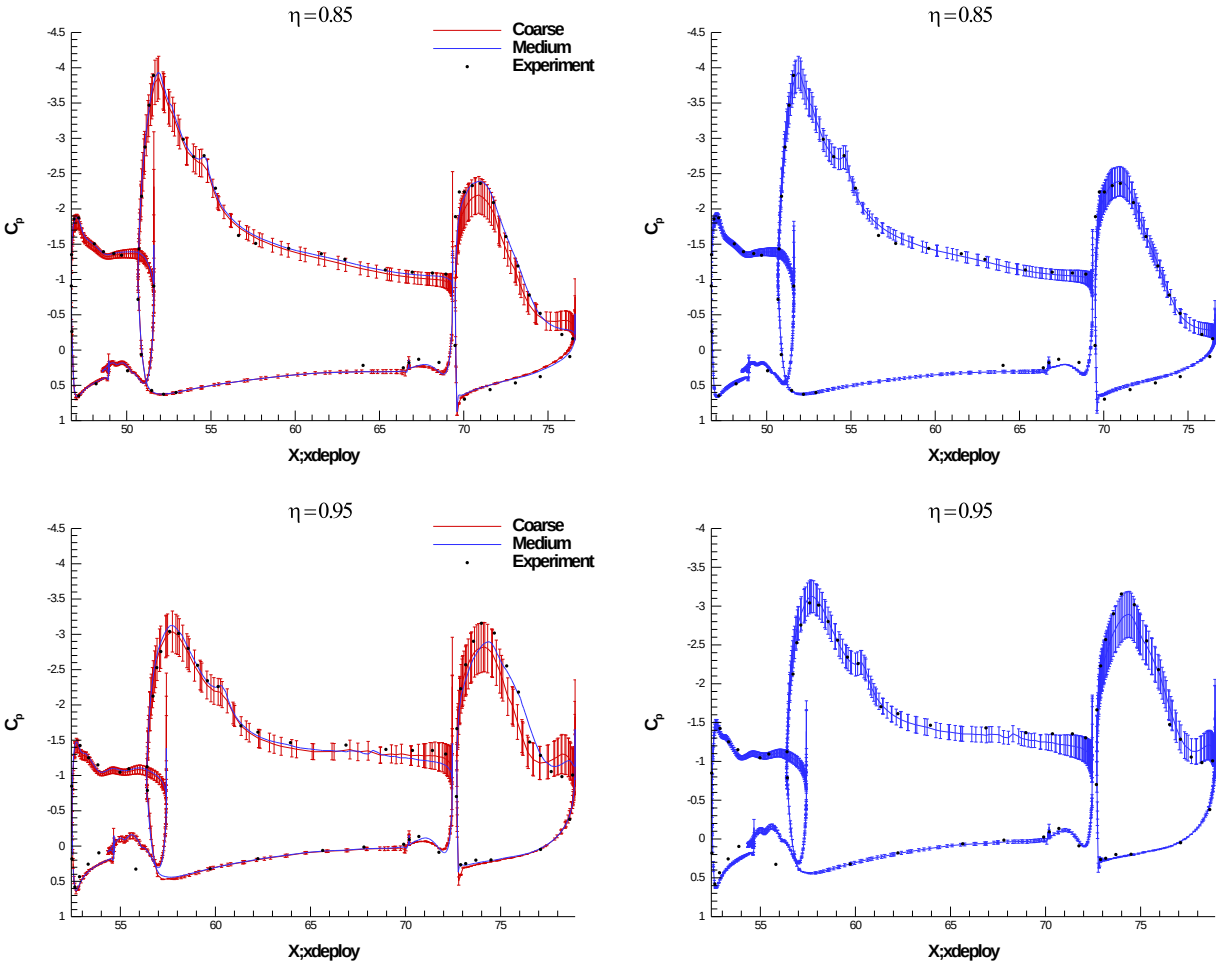
USM3D Results, α=13° (2)



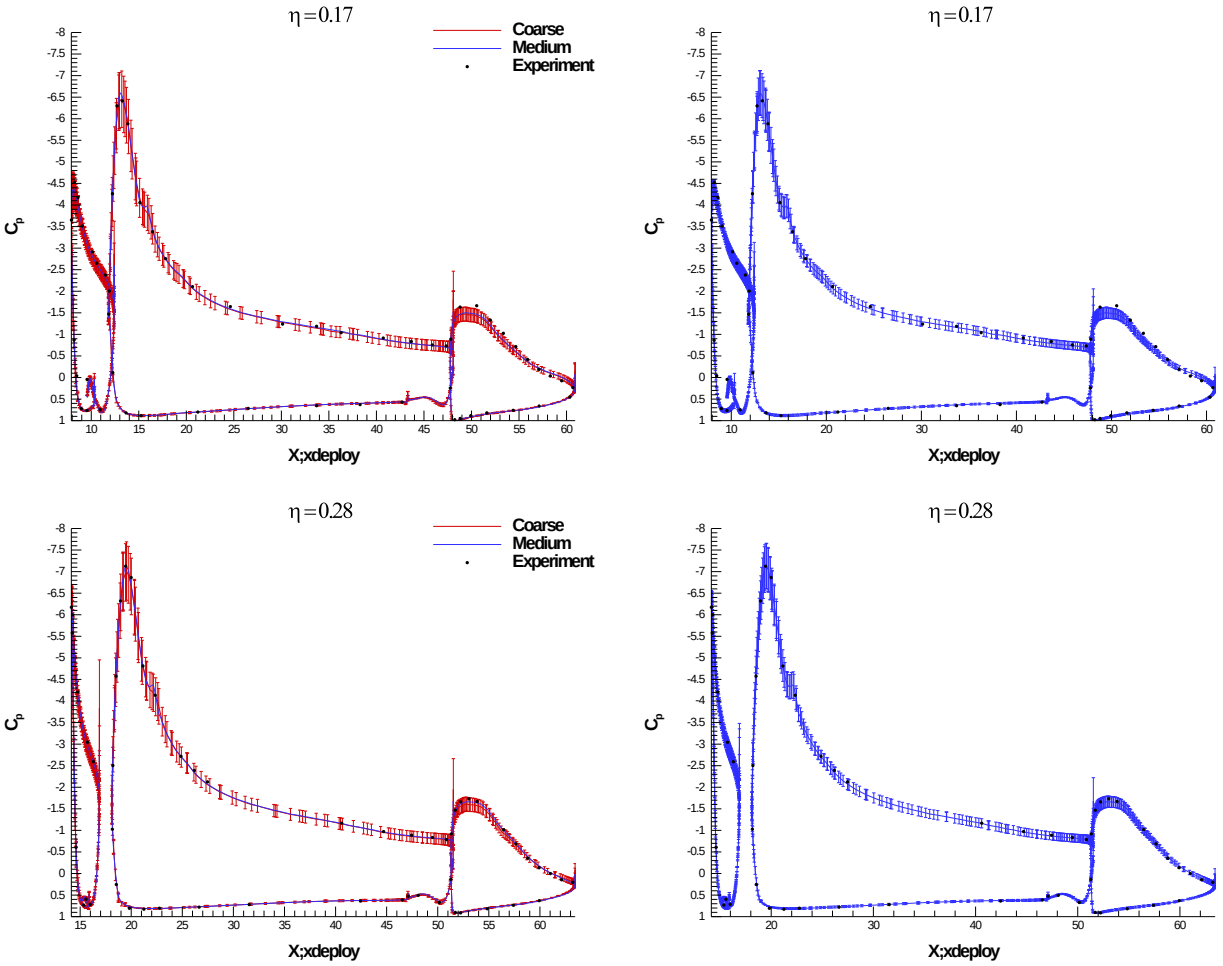
USM3D Results, α=13° (3)



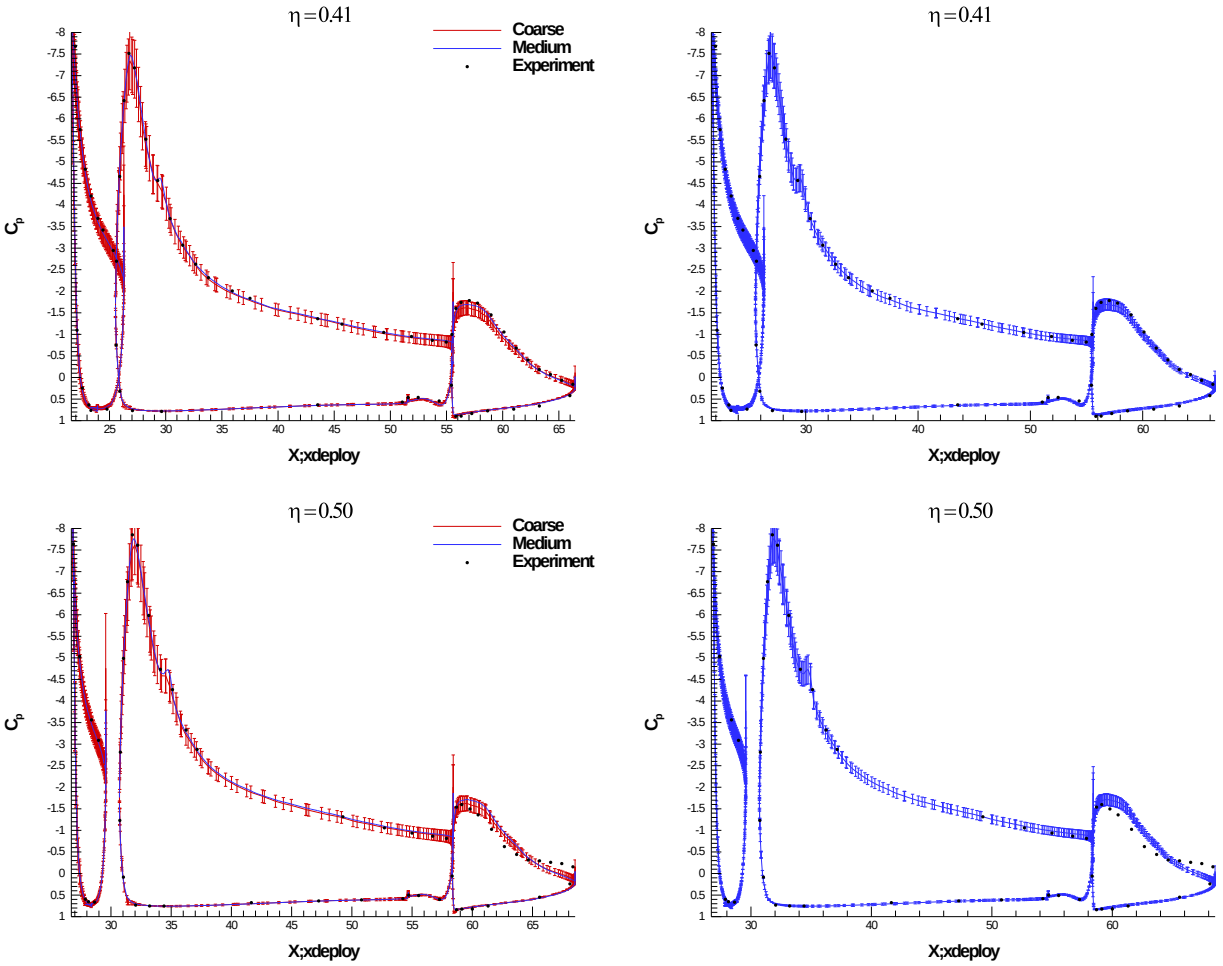
USM3D Results, α=13° (4)



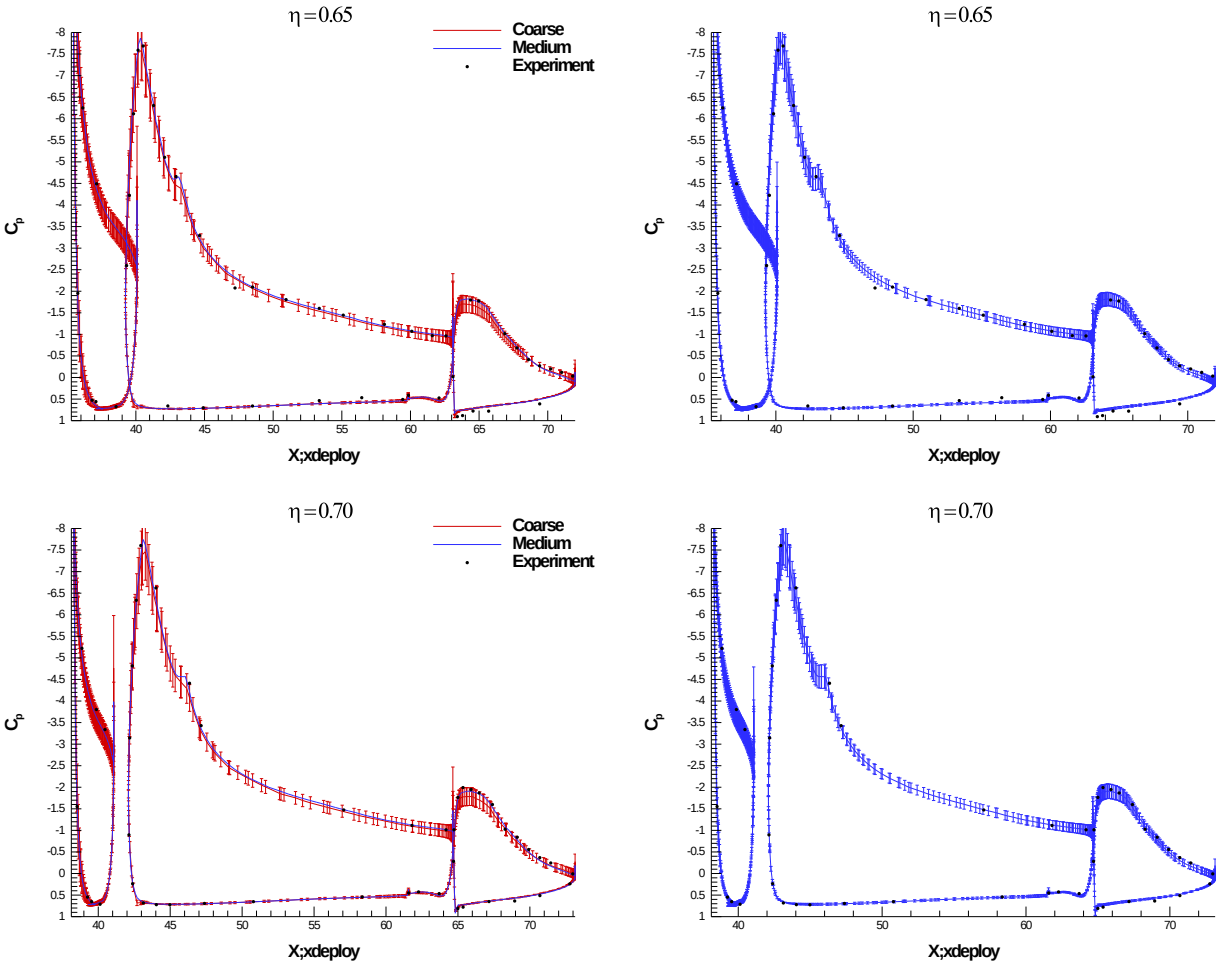
USM3D Results, α=28° (1)



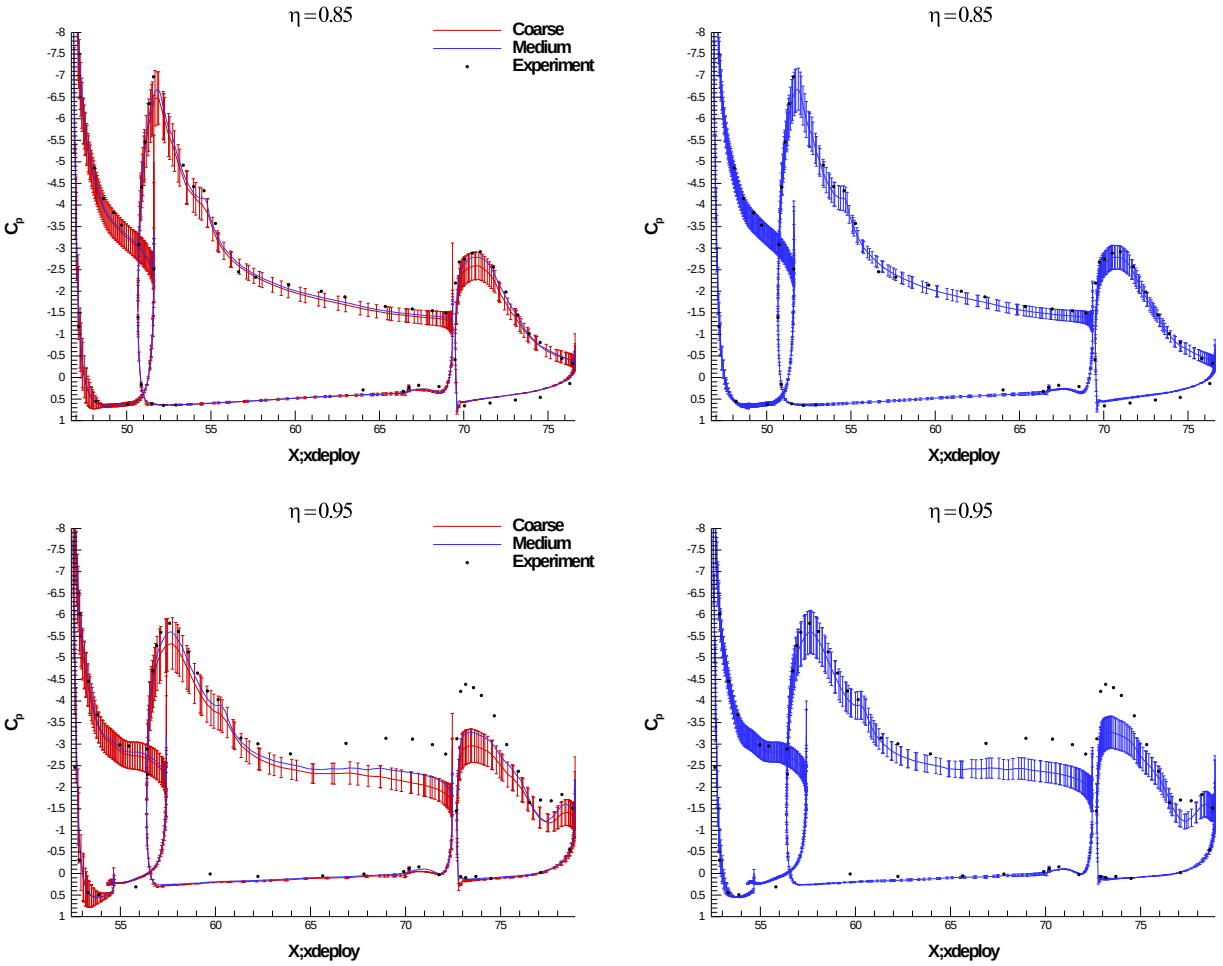
USM3D Results, α=28° (2)



USM3D Results, α=28° (3)

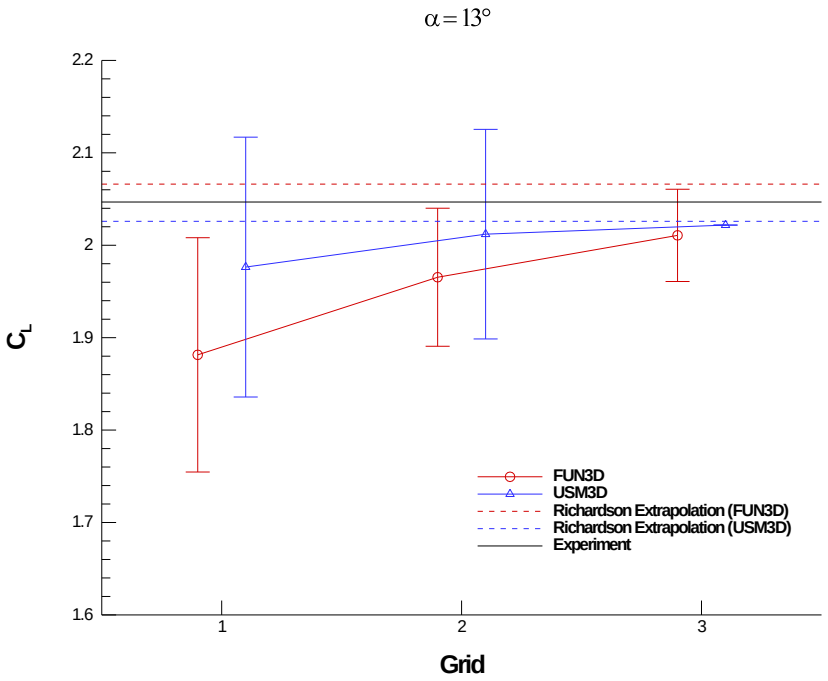


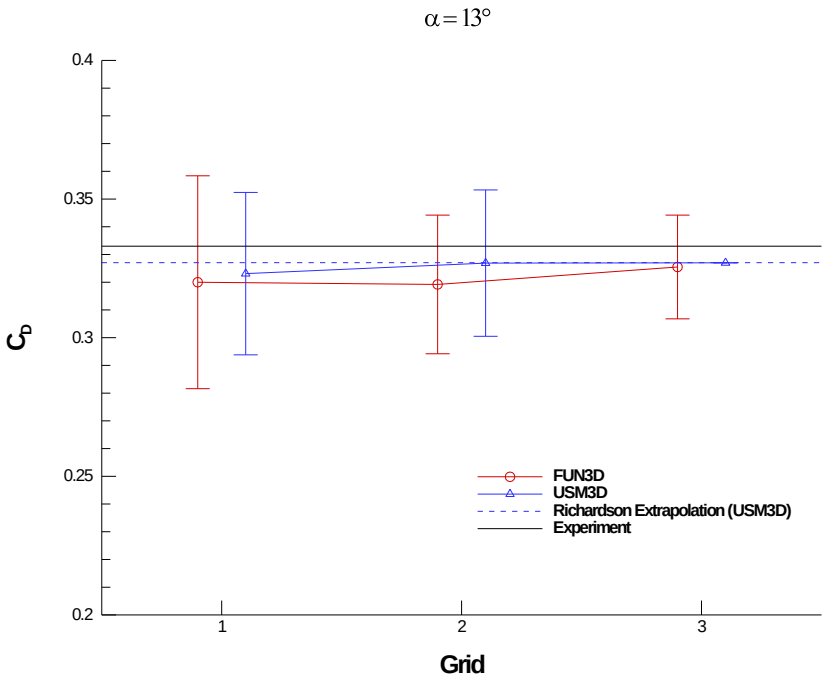
USM3D Results, α=28° (4)



Lift/Drag Results, α=13°

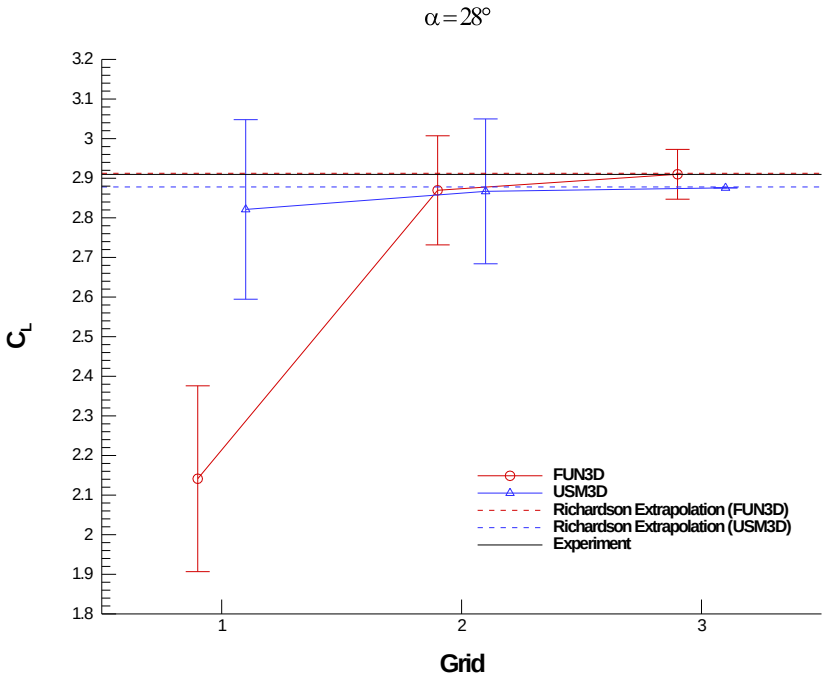
- Error bars for *FUN3D* solution are reasonable for C_L, very conservative for C_D
- Error bars for *USM3D* solution overly conservative for both C_L, C_D

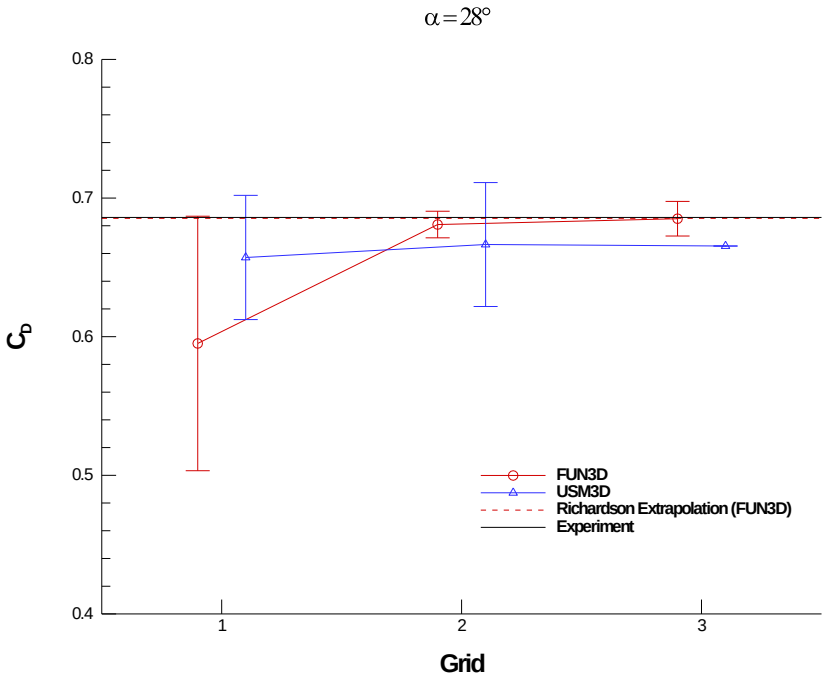




Lift/Drag Results, α=28°

- Error bars for FUN3D solution are improved for C_L and favorable for C_D
- Error bars for USM3D solution again rather large, far exceeding coarse-to-fine grid increment
- Remains to be resolved





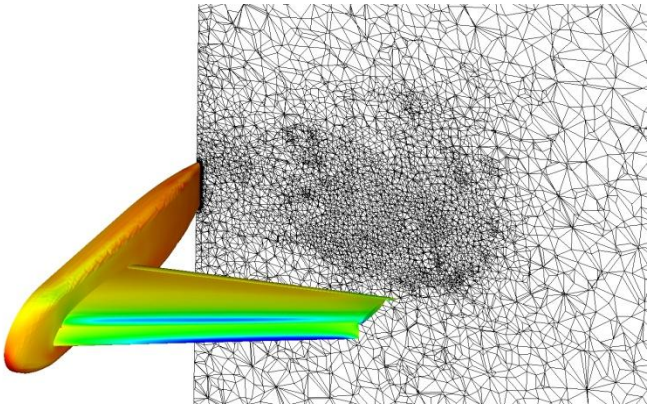
Error Reduction – CRISP CFD

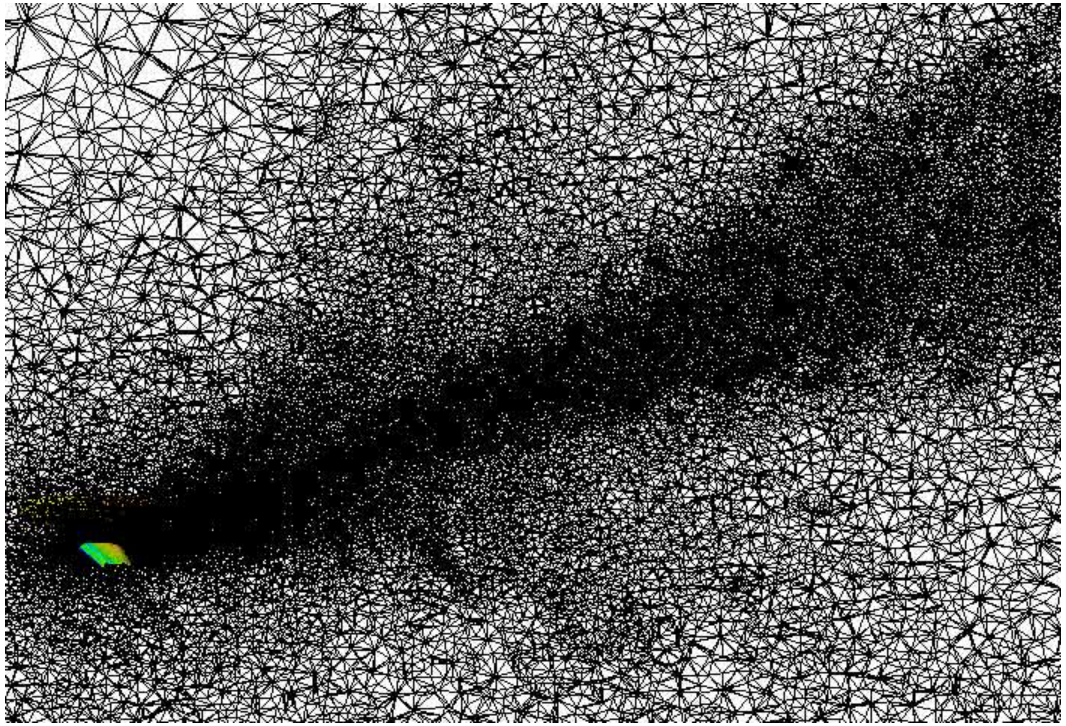
- Can *FUN3D* Coarse Grid predictions be improved via mesh adaptation?
- Adaptation criterion based on sources/residuals of ETE
  - Premise: eliminate/minimize the source, eliminate where error appears
  - Adapted grid still ½ the size of medium *FUN3D* grid
- Use of parallel unstructured grid adaptation package for performing mesh modifications and working with adapted meshes*
  - Conforming mesh modification operations for mixed element grids
  - Supports grids from *Gridgen*, *VGRIDns*, *SolidMesh*, *ICEM Tetra*, *etc*.
  - Compatible with *AVUS/Cobalt*, *FUN3D*, *USM3D*
- Modules available for performing:
  - Mesh coarsening and refinement
  - Load rebalancing and interprocessor communication updates
  - Error estimation and solution interpolation

* Cavallo, P.A., and Grismer, M.J., "A Parallel Adaptation Package for Three-Dimensional Mixed-Element Unstructured Meshes," *Journal of Aerospace Computing, Information, and Communication*, Vol. 2, No. 11, pp. 433-451, 2005.

Adapted Grid

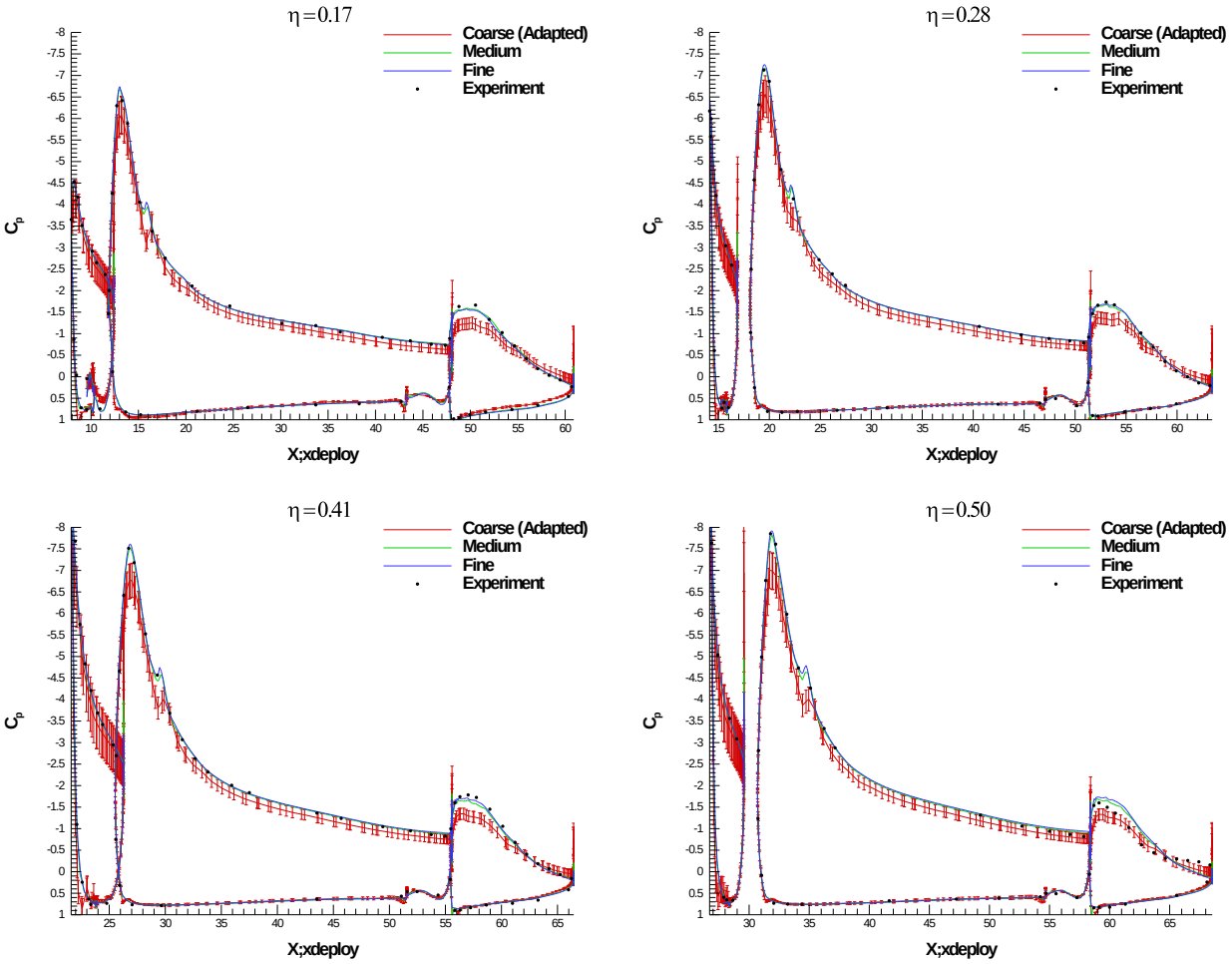
- Viscous residual term targeted downstream wake (error in μ_t)
- Inviscid residual term dominated near field refinement
- Adapted grid between coarse and medium in number of vertices



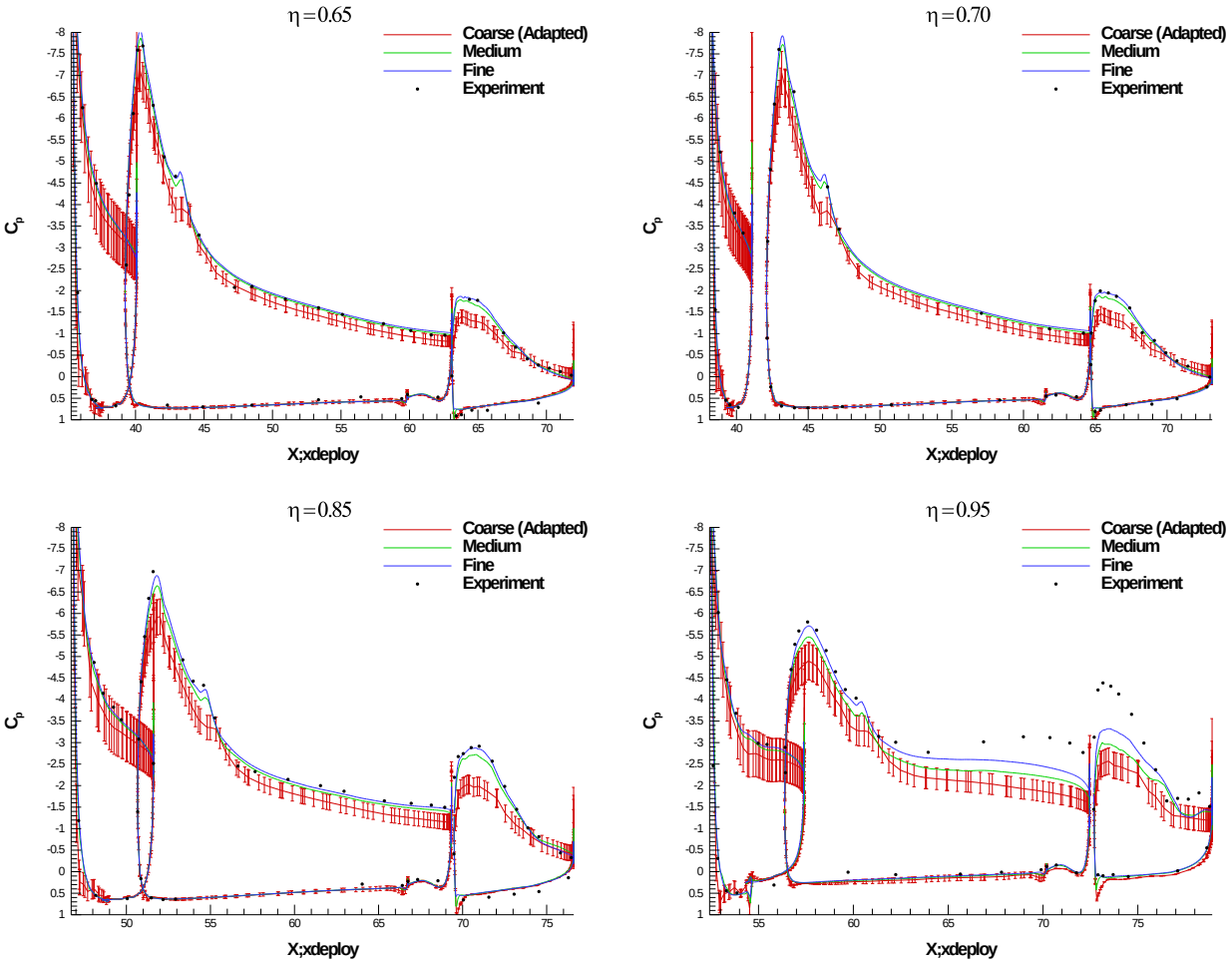


- Circulation over wing improved and flow now attached
- C_L increased to 2.66
- Error predictions also improved

Adapted FUN3D Results, α=28° (1)



Adapted FUN3D Results, α=28° (2)



Conclusions and Lessons Learned

- In general, predicted error bars for C_p were consistent in containing fine grid results for both flow solvers
- Predicted error bars for C_L and C_D generally too conservative
- Error predictions are only suitable if grid is within monotonic range
  - ETE can not predict presence/absence of flow features (*e.g.* separation)
  - Error solution obtained on same grid as flow solution, subject to same discretization errors
  - How to determine if particular grid/solution is in monotonic range?
- Mesh adaptation improved poor *FUN3D* CFD prediction and ETE prediction, but still missed suction peak on flap
- Suitability of node-centered ETE solver for cell-centered CFD solutions?

➢ ***Methods require thorough validation*** to establish confidence in use: need to continue pushing the envelope with real applications

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- ***Questions?***

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