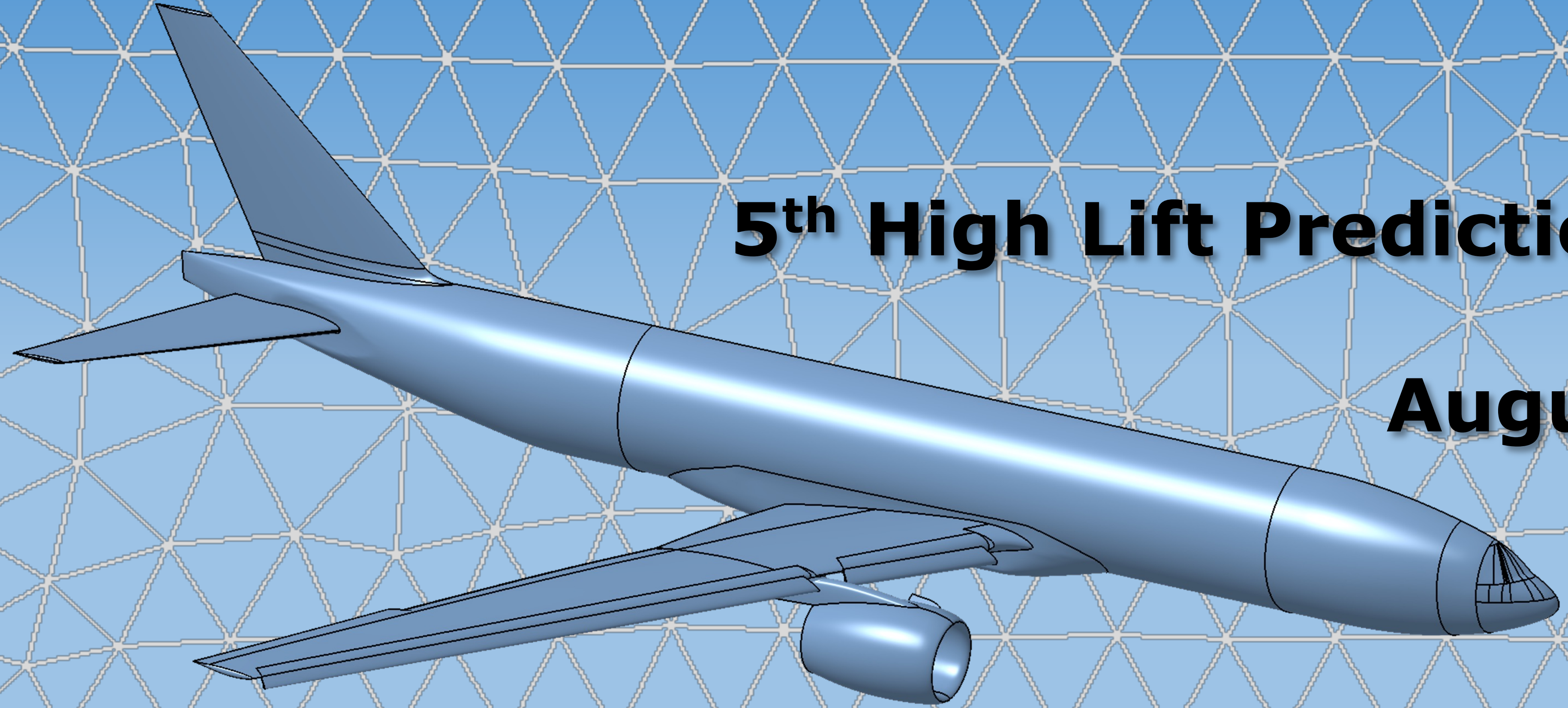

5th High Lift Prediction Workshop

August 2-3, 2024

High-Order TFG

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Steve Karman
Oak Ridge National Laboratories

Special thanks to:

- High-Order TFG Participants
  - Meshing: Vangelis Skaperdas, Xevi Roca and Eloi Ruiz
  - Solver: Kevin Holst, and ZJ Wang
- HLPW5 Leadership team and TFG Ideas

Key Questions for High-Order TFG

- Can high-order meshes be generated for a high-lift configuration?
  - What are the meshing requirements?
  - Does high-order mean less mesh sensitivity?

- Can asymptotic solutions be achieved with high-order methods?
  - ✓RANS in 2D
  - ✗RANS in 3D
  - WMLES in 3D

Outline

- Terminology
  - What is a curved mesh?
- An academic example: Optimal adapted mesh distributions
- Multi-Element Airfoil: High-Lift 4 Verification
- Challenges Curving Meshes in 3D
- Submitted Solutions to High-Order TFG
- Summary

Terminology

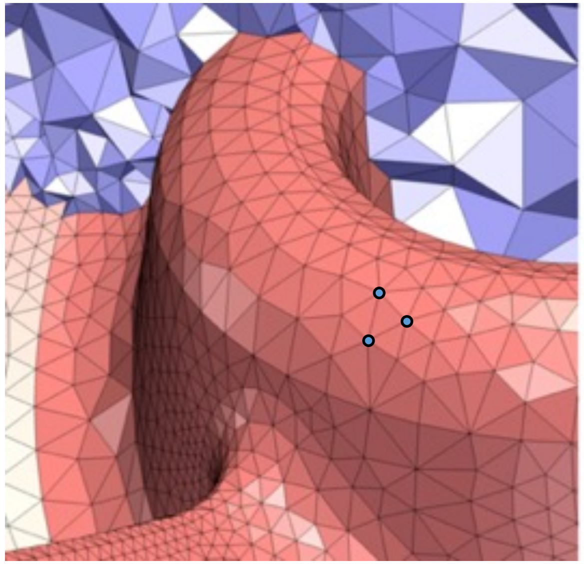
- Finite Elements: Elemental polynomial mesh and solution
  - Order of accuracy by the polynomial degree

- Mesh order
  - Q1 (linear, e.g. standard mesh)
  - Q2 (quadratic)
  - Q3 (cubic)
  - Q4 (quartic)

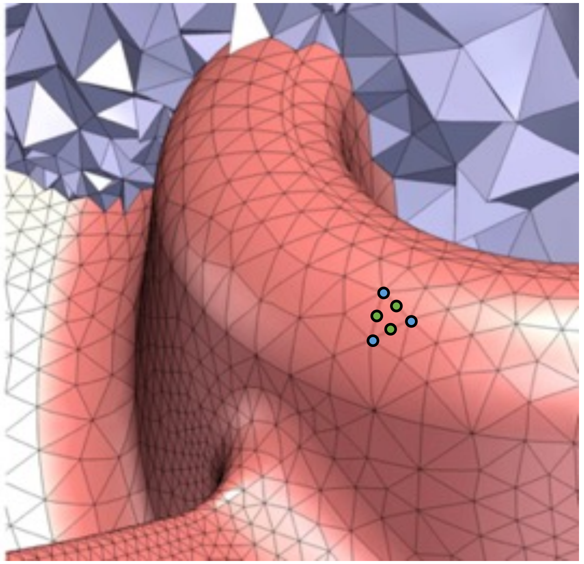
- Solution order
  - P1 is 2nd order solution
  - P2 is 3rd order solution
  - P3 is 4th order solution

Linear vs. Curved Meshes

- **Linear mesh**: linear (Q1) approximation to curved geometry
- **Curved mesh**: features explicit curvature (high-order polynomial element representation)
  - Mesh elevation: Q1 → Q2 → Q3



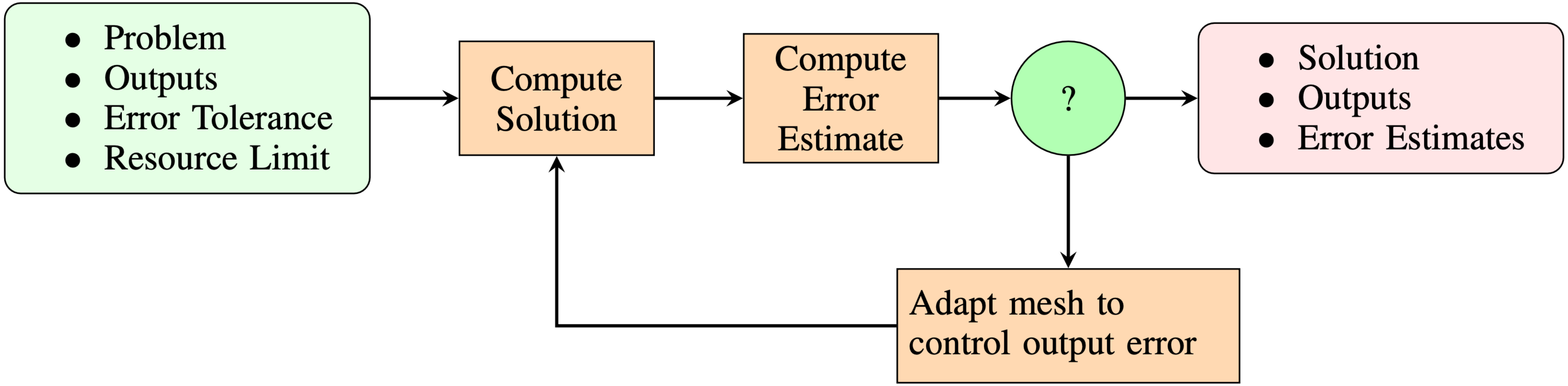
Straight-edged mesh



Curved mesh

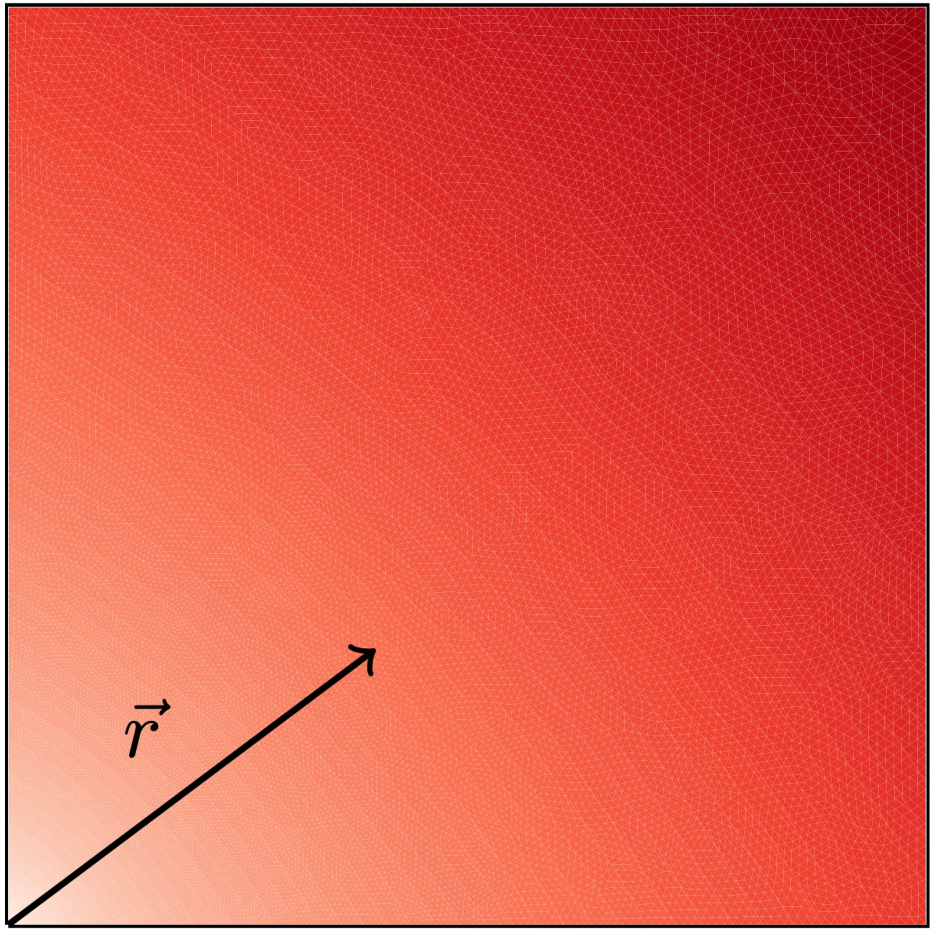
- **Beneficial in applications sensitive to curvature**:
  - Fluid dynamics
  - Mechanics
  - Electromagnetics
  - …

High-Order Output Based Mesh Adaptation



- Error estimates via Dual Weighted Residual (Adjoint) method
- Mesh Optimization:
  - Find mesh that minimizes output error for a target Degree of Freedom (DOF) count
- MOESS: Provably optimal meshes (Hugh Carson, PhD Dissertation, MIT 2020)

An Academic Example: r^{α}-type Corner Singularity



- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^{\alpha}\sin(\alpha(\theta+\theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

$$u_{h,p} = \arg\min_{v_{h,p}} \|u - v_{h,p}\|^2_{L^2(\Omega)}$$

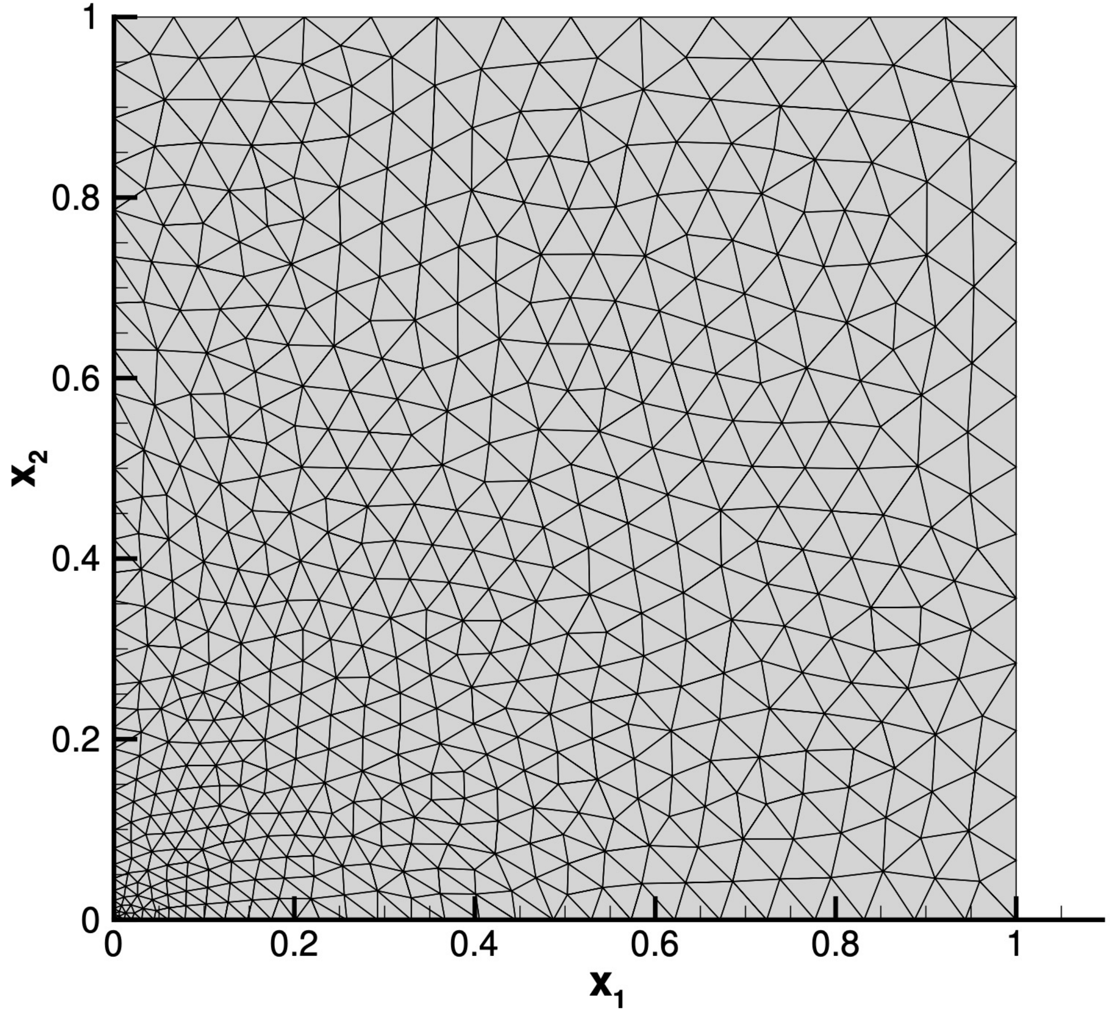
$$\mathcal{E} = \sqrt{\int_{\Omega}(u-u_{h,p})^2 dx}$$

- Analytic spacing distribution:

$$h = C_r r^{k_a}, \quad k_a = 1 - \frac{\alpha+1}{p+2}$$

- Optimal distribution changes with p

An Academic Example: r^α-type Corner Singularity



P1, 4000 DOF, 1333 Elem

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^\alpha \sin(\alpha(\theta + \theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

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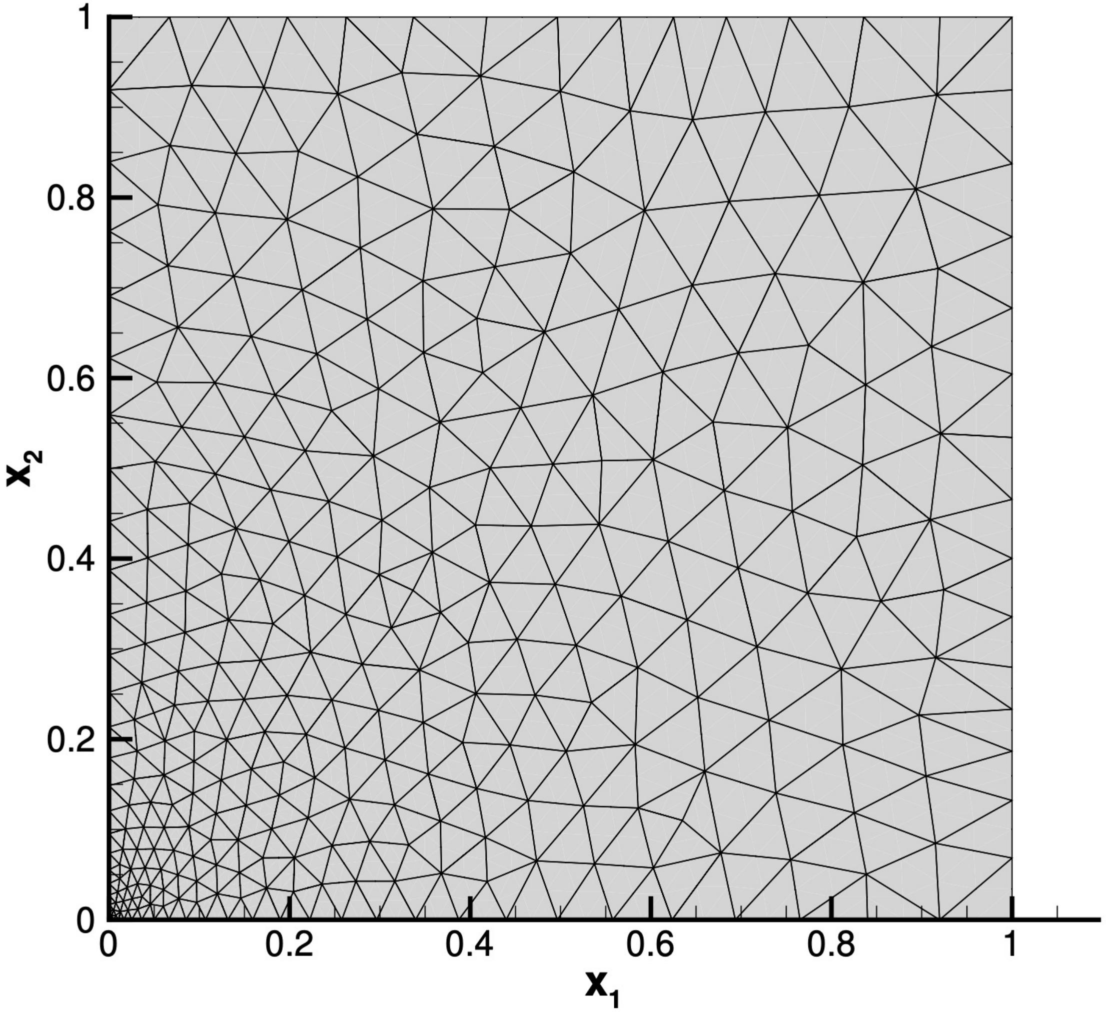
- Optimal distribution changes with p

An Academic Example: r^{α}-type Corner Singularity



P2, 4000 DOF, 667 Elem

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^{\alpha}\sin(\alpha(\theta+\theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

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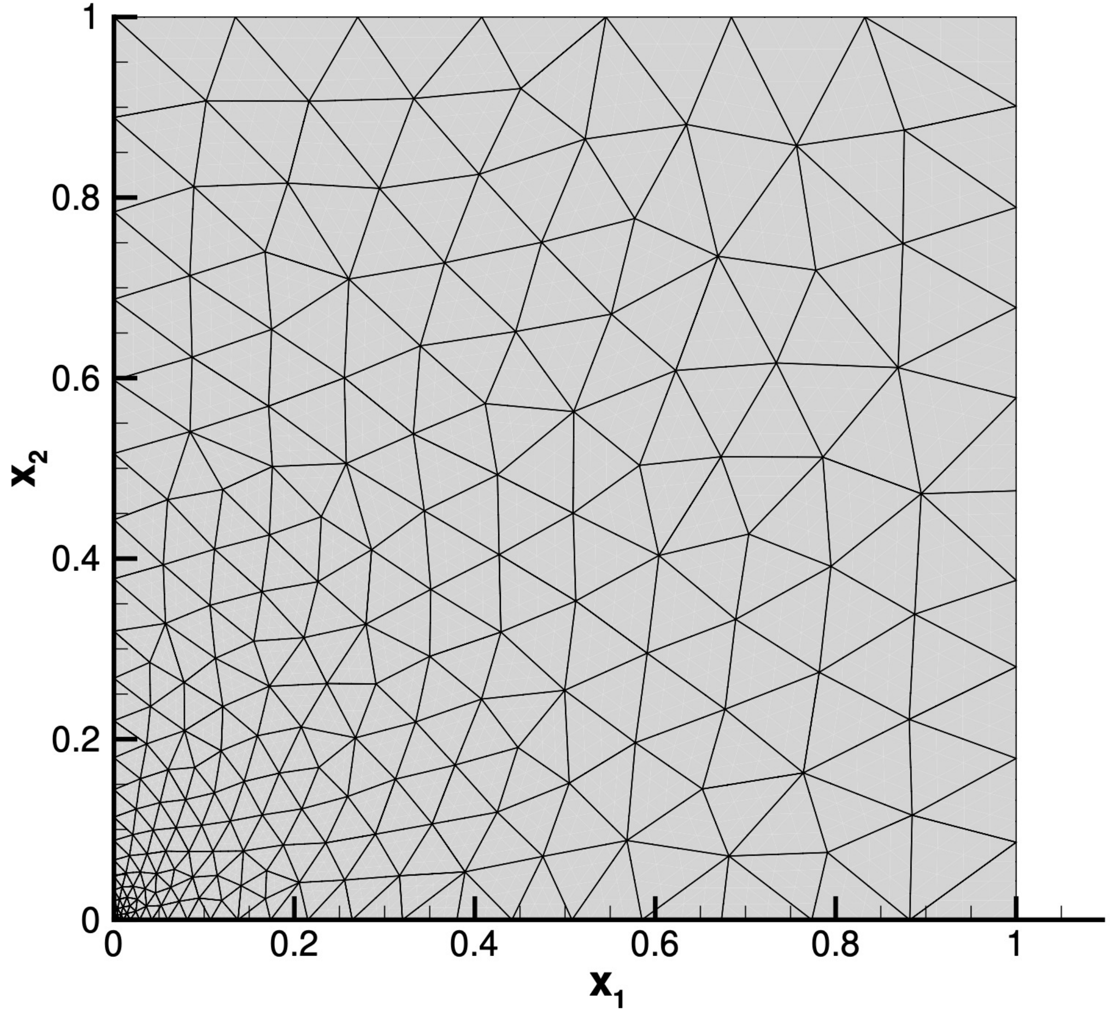
- Optimal distribution changes with p

An Academic Example: r^α-type Corner Singularity



P3, 4000 DOF, 400 Elem

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^\alpha \sin(\alpha(\theta + \theta_0)), \ \alpha = \frac{2}{3}, \ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

$$u_{h,p} = \underset{v_{h,p}}{\arg\min} \|u - v_{h,p}\|^2_{L^2(\Omega)}$$

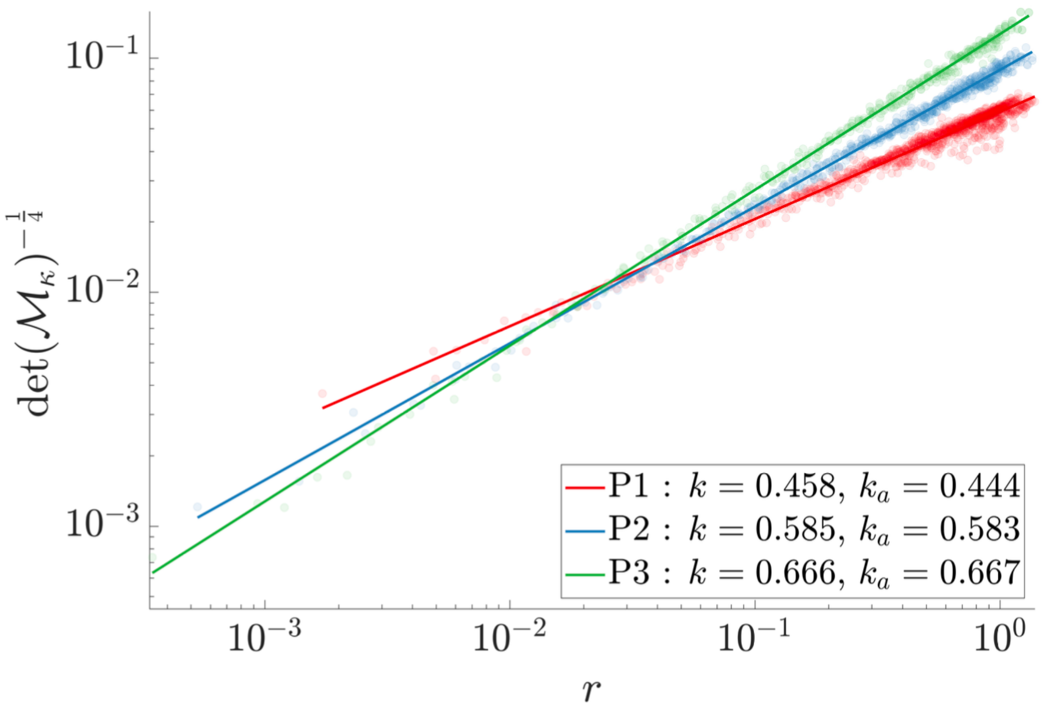
$$\mathcal{E} = \sqrt{\int_\Omega (u - u_{h,p})^2 dx}$$

- Analytic spacing distribution:

$$h = C_r r^{k_a}, \quad k_a = 1 - \frac{\alpha + 1}{p + 2}$$

- Optimal distribution changes with p

An Academic Example: r^α-type Corner Singularity



MOESS 'discovers' analytic spacing distribution

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^\alpha \sin(\alpha(\theta+\theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

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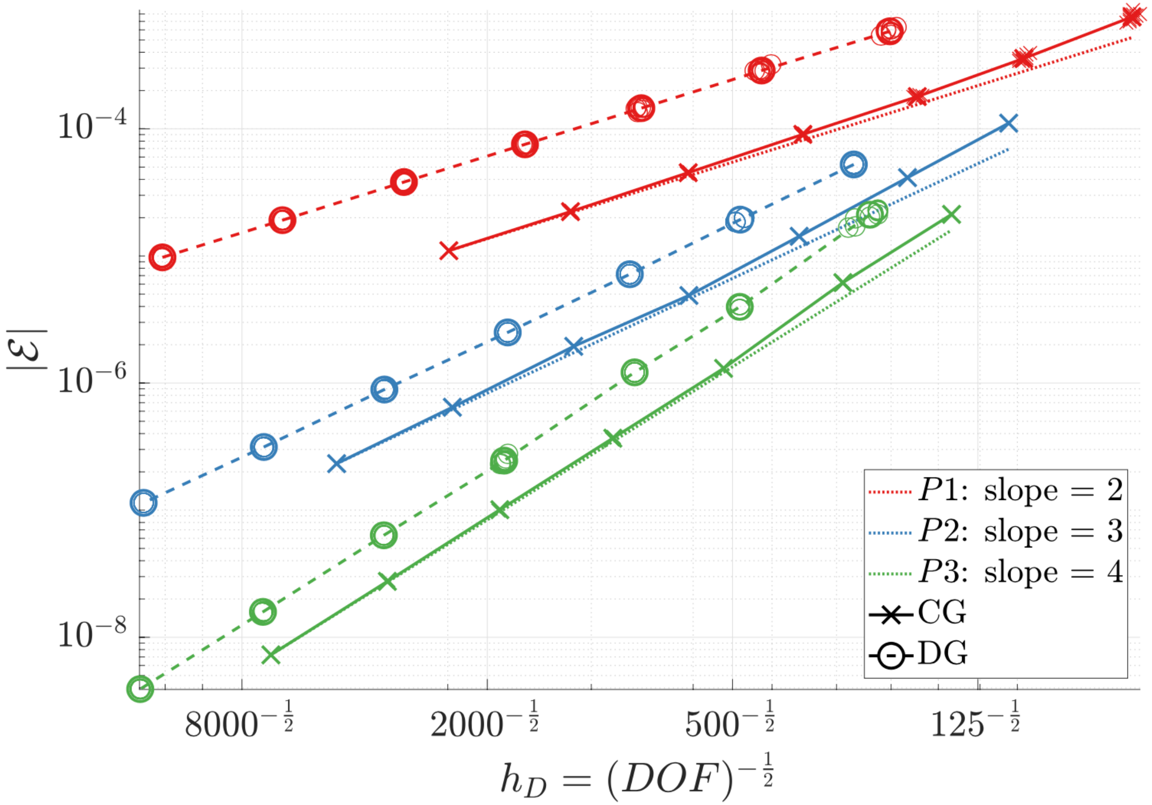
An Academic Example: r^α-type Corner Singularity



MOESS recovers order of accuracy $O(h^{p+1})$

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^\alpha \sin(\alpha(\theta+\theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

$$u_{h,p} = \underset{v_{h,p}}{\arg\min} \|u - v_{h,p}\|^2_{L^2(\Omega)}$$

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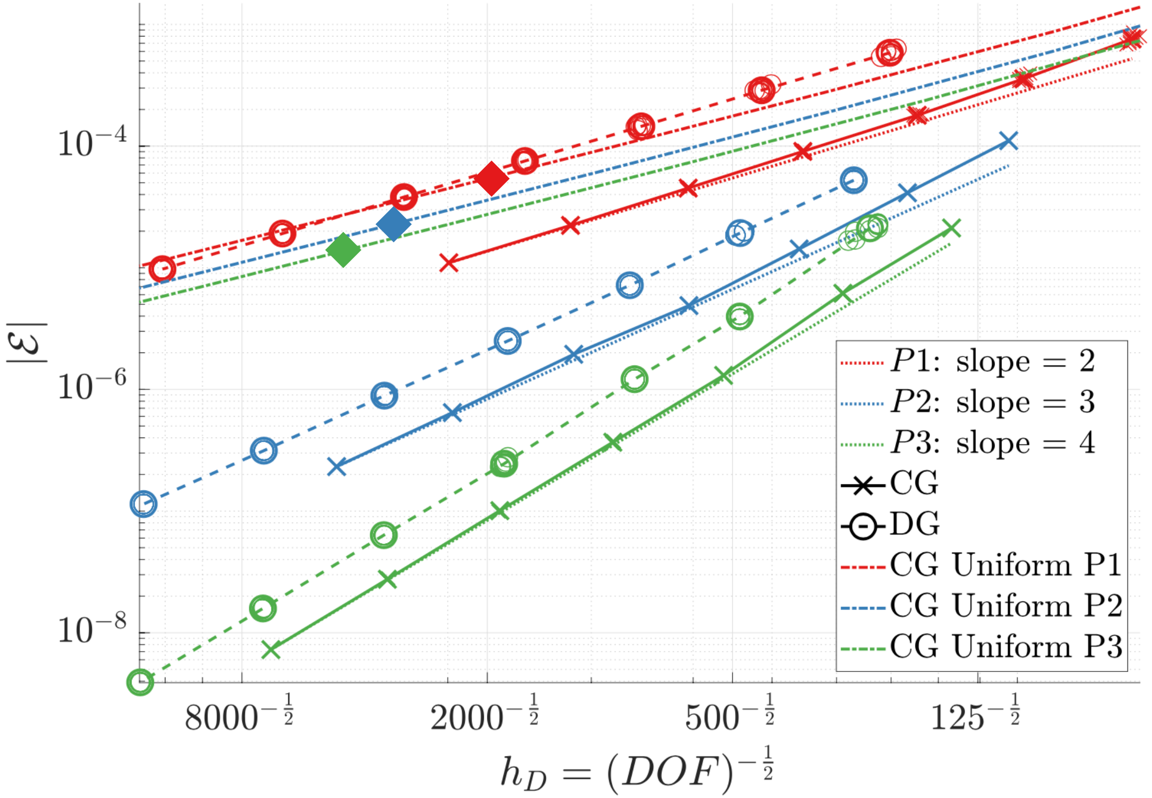
An Academic Example: r^α-type Corner Singularity



Uniform meshes converge at $O(h^{\frac{5}{3}})$ for all p

Fixed mesh P-convergence only works for smooth functions

- Mimics trailing edge pressure singularity:

$$u(r,\theta) = r^\alpha \sin(\alpha(\theta + \theta_0)),\ \alpha = \frac{2}{3},\ \theta_0 = \frac{\pi}{2}$$

- L^2 projection and interpolation error:

$$u_{h,p} = \arg\min_{v_{h,p}} \|u - v_{h,p}\|^2_{L^2(\Omega)}$$

$$\mathcal{E} = \sqrt{\int_\Omega (u - u_{h,p})^2 dx}$$

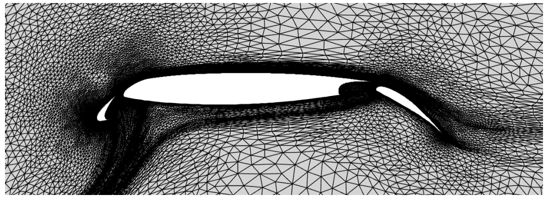
- Analytic spacing distribution:

$$h = C_r r^{k_a}, \quad k_a = 1 - \frac{\alpha + 1}{p + 2}$$

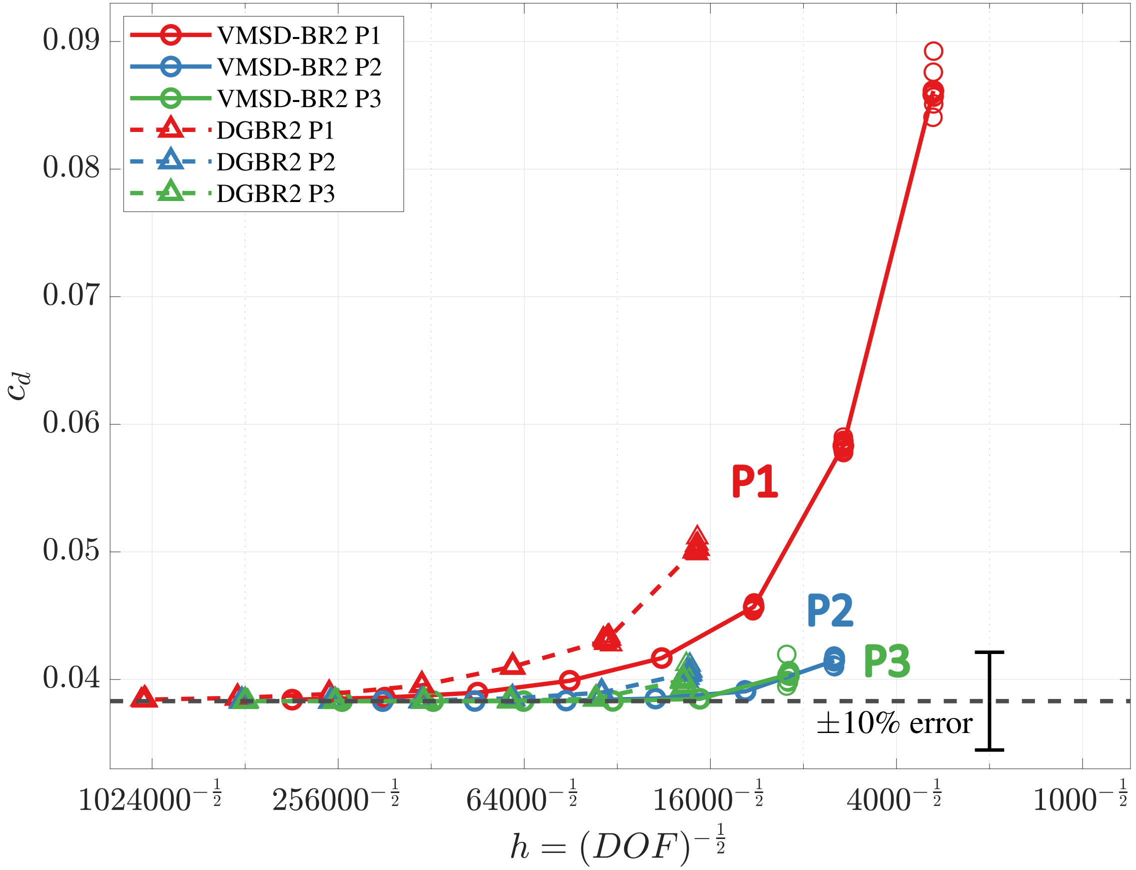
- Optimal distribution changes with p

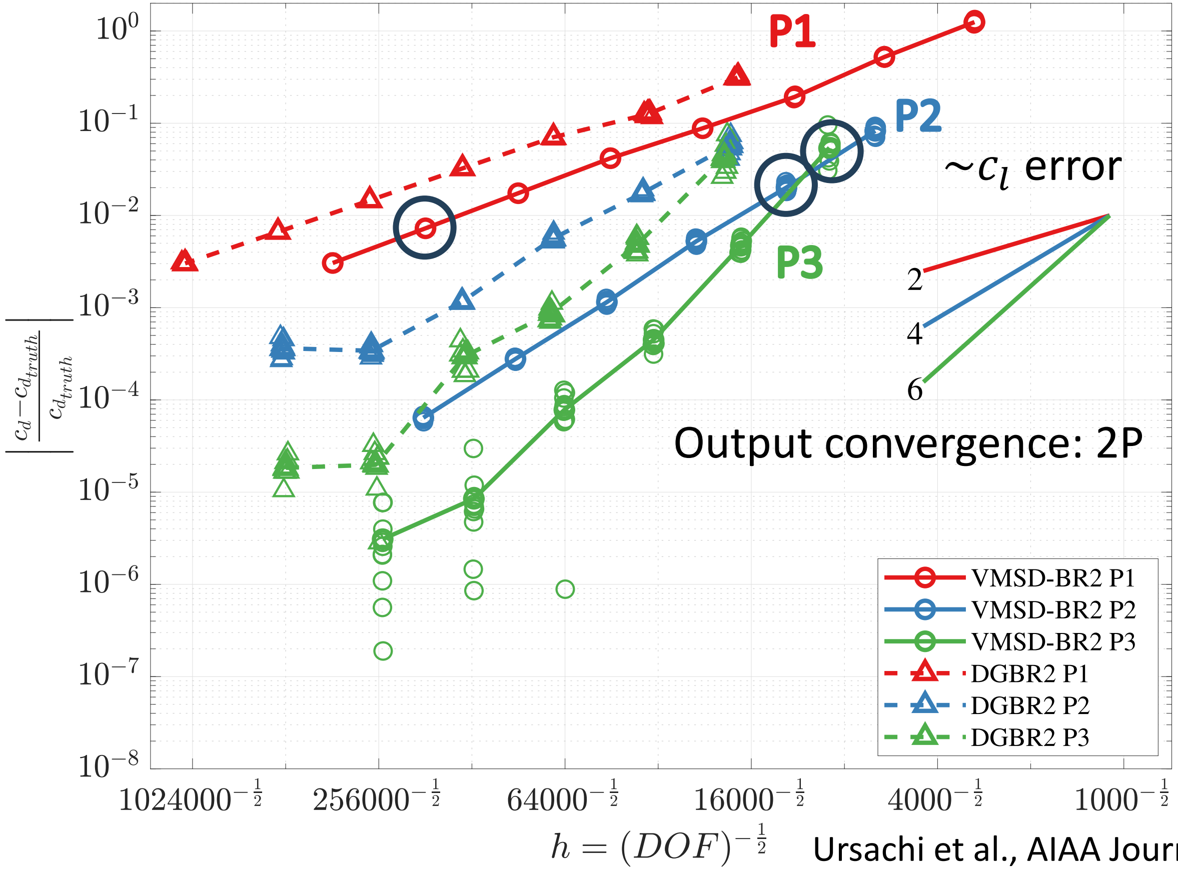
Multi-Element High-Lift Airfoil

High-Lift 4 Verification Case



AoA 8° Adapted Q3 Meshes

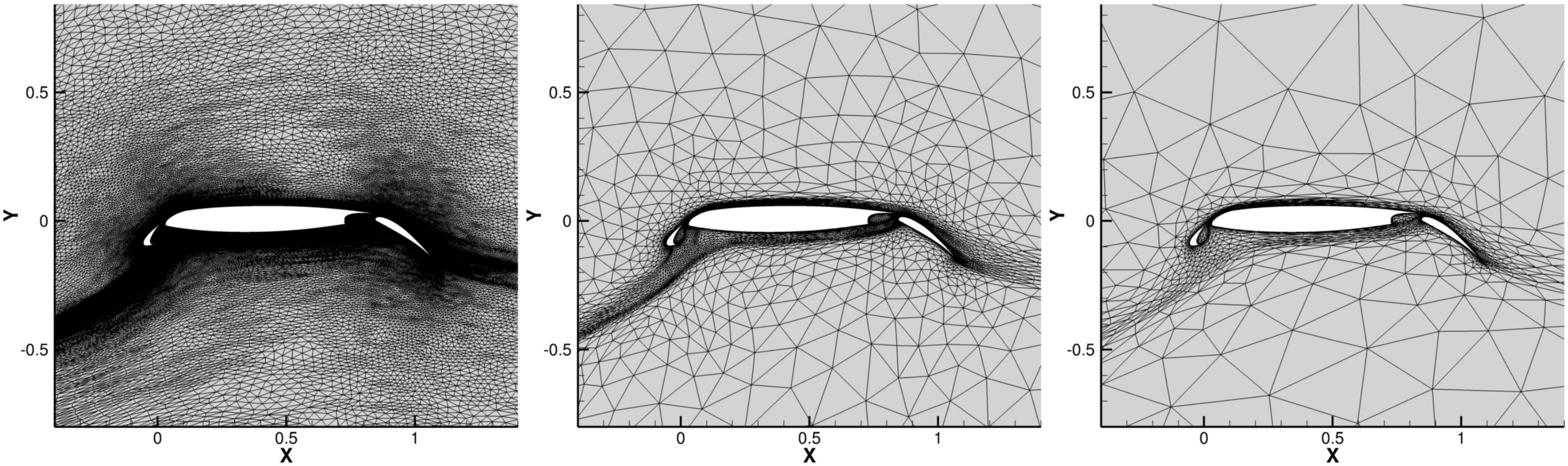




Ursachi et al., AIAA Journal, 2021

- Adaptation obtains optimal convergence rates
  - Uncovers potential of high-order methods

$\alpha = 8°$ Q3 Meshes at 10^{-4} c_l Error Level

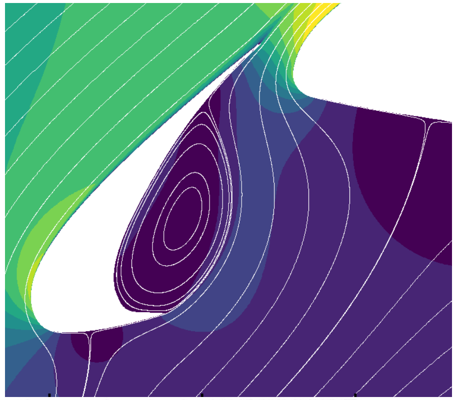


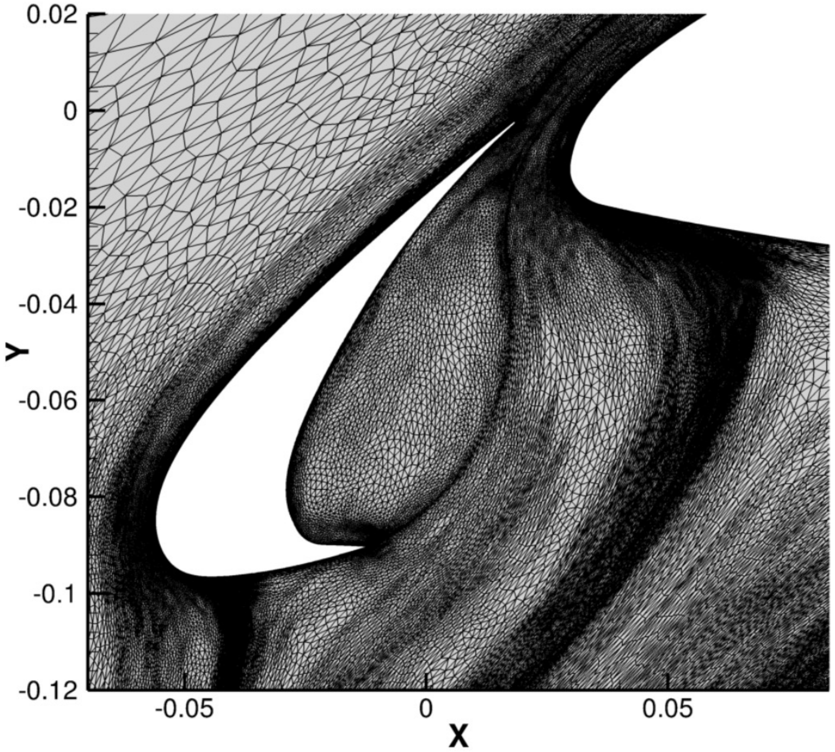
P1 341k DOF
c_l error: 7.8×10^{-5}

P2 42k DOF
c_l error: 3.5×10^{-5}

P3 29k DOF
c_l error: 1.5×10^{-5}

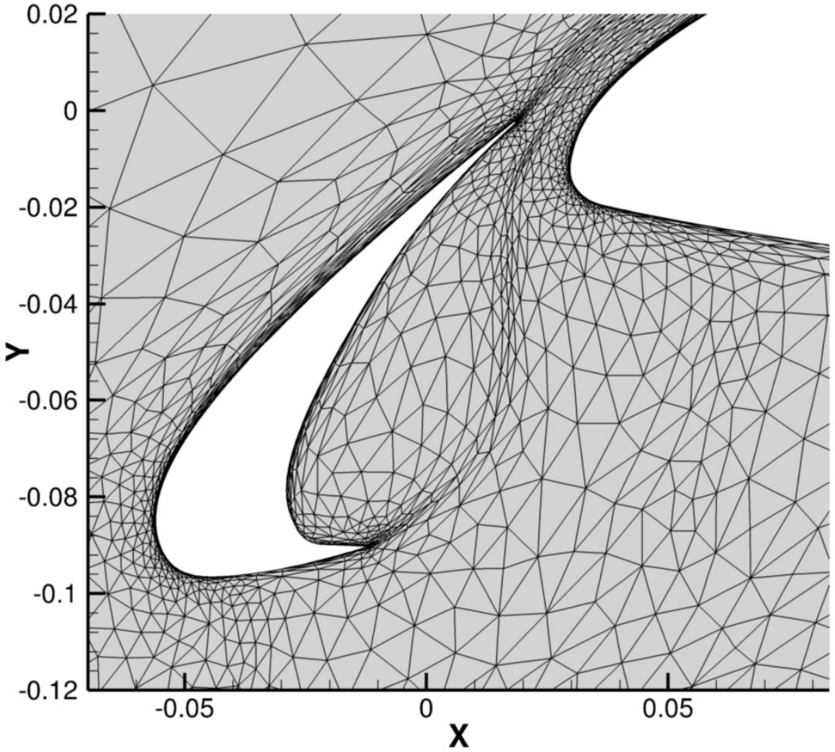
$\alpha = 8^\circ$ Q3 Meshes at 10^{-4} c_l Error Level





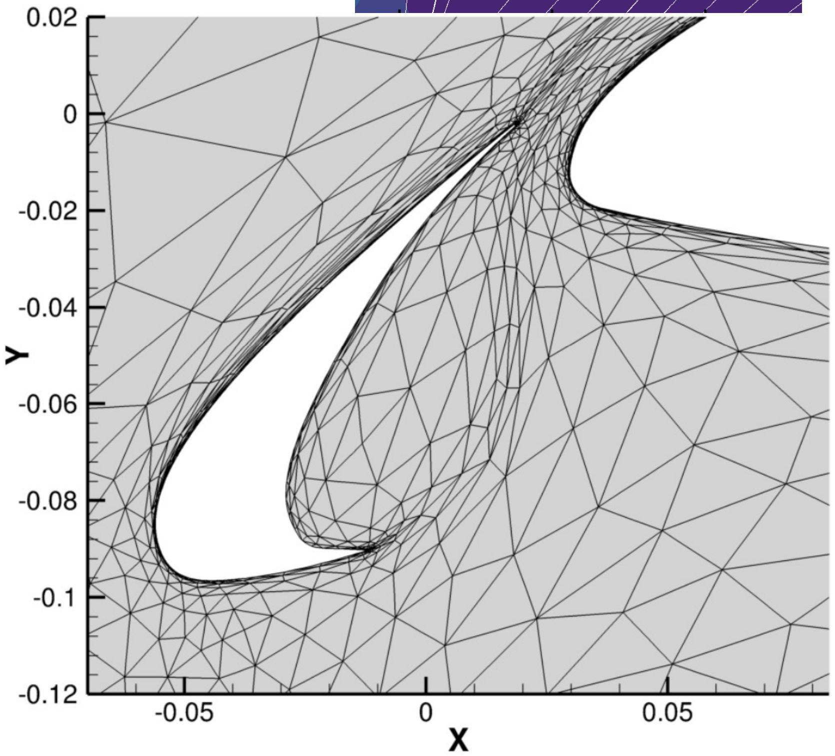
P1 341k DOF

c_l error: 7.8×10^{-5}



P2 42k DOF

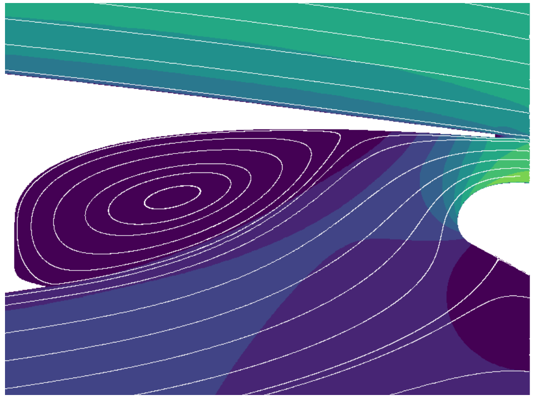
c_l error: 3.5×10^{-5}

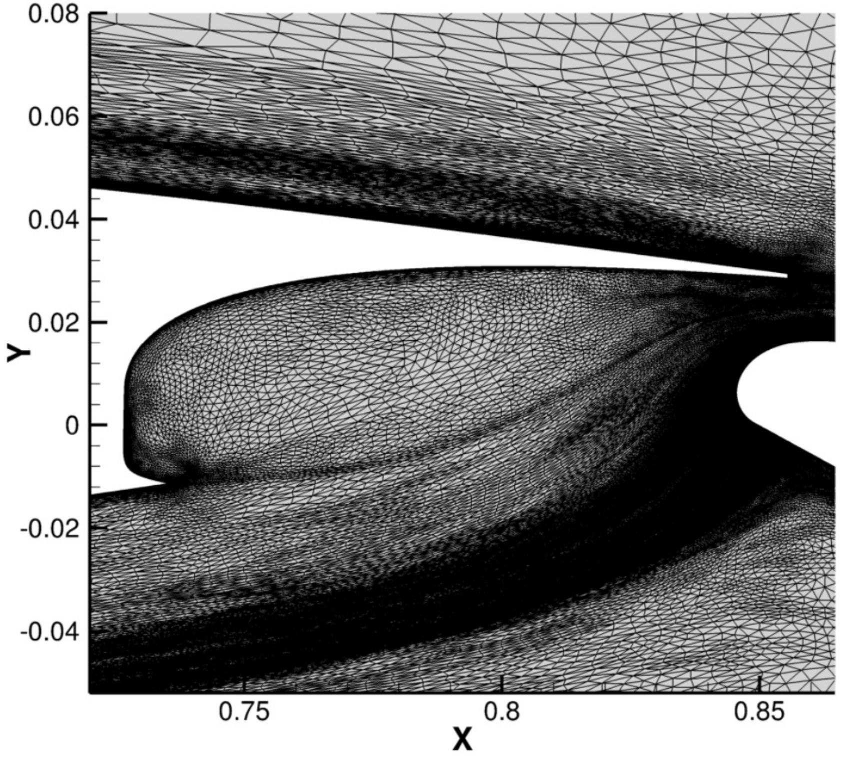


P3 29k DOF

c_l error: 1.5×10^{-5}

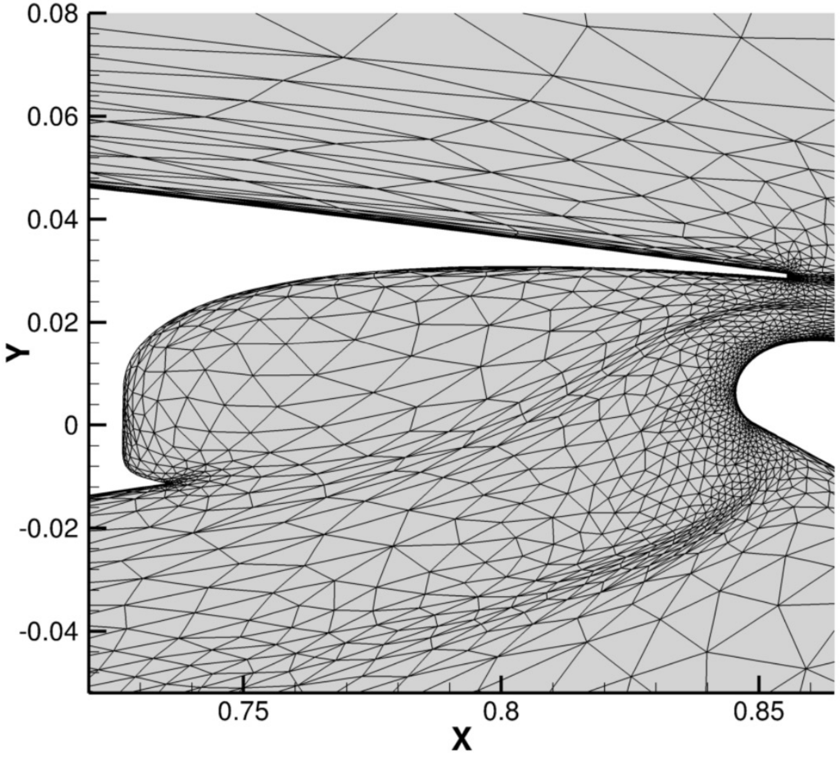
$\alpha = 8^\circ$ Q3 Meshes at 10^{-4} c_l Error Level





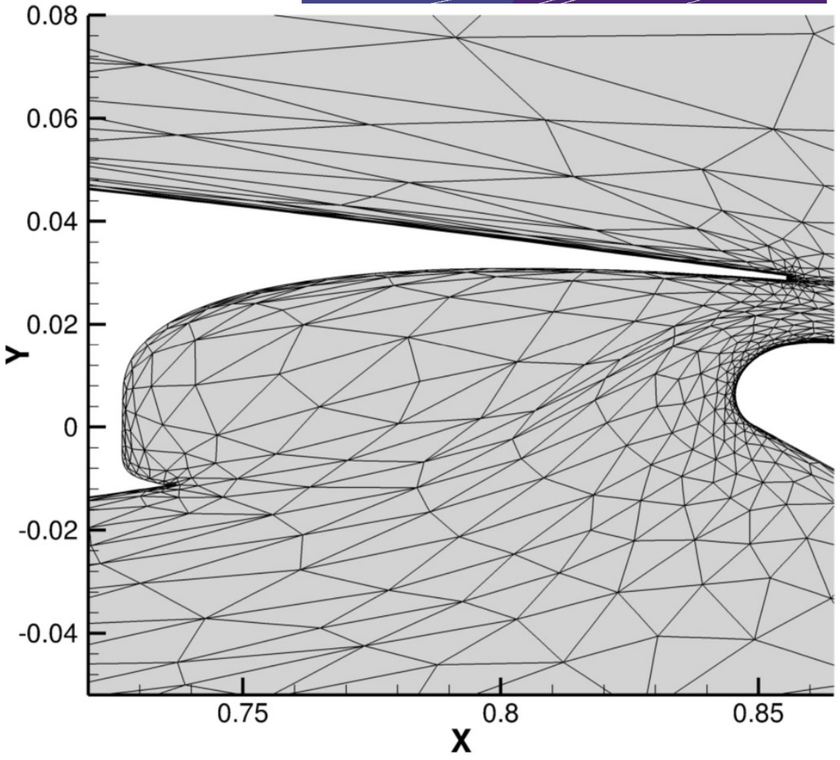
P1 341k DOF

c_l error: 7.8×10^{-5}



P2 42k DOF

c_l error: 3.5×10^{-5}



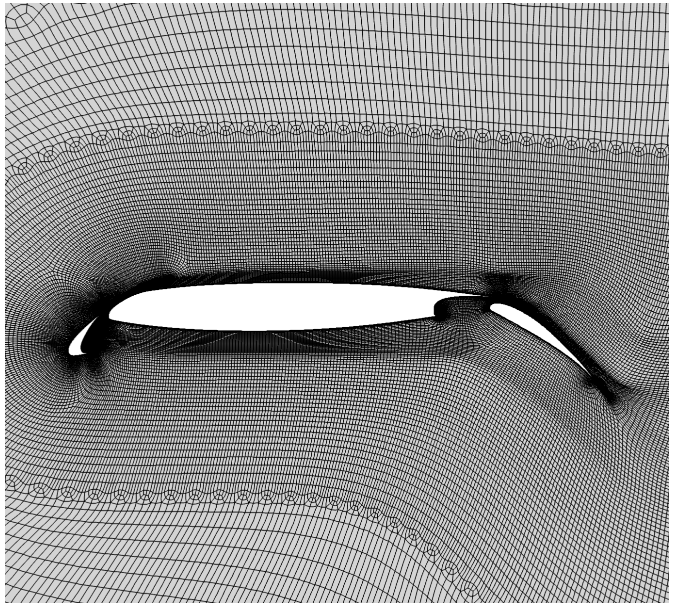
P3 29k DOF

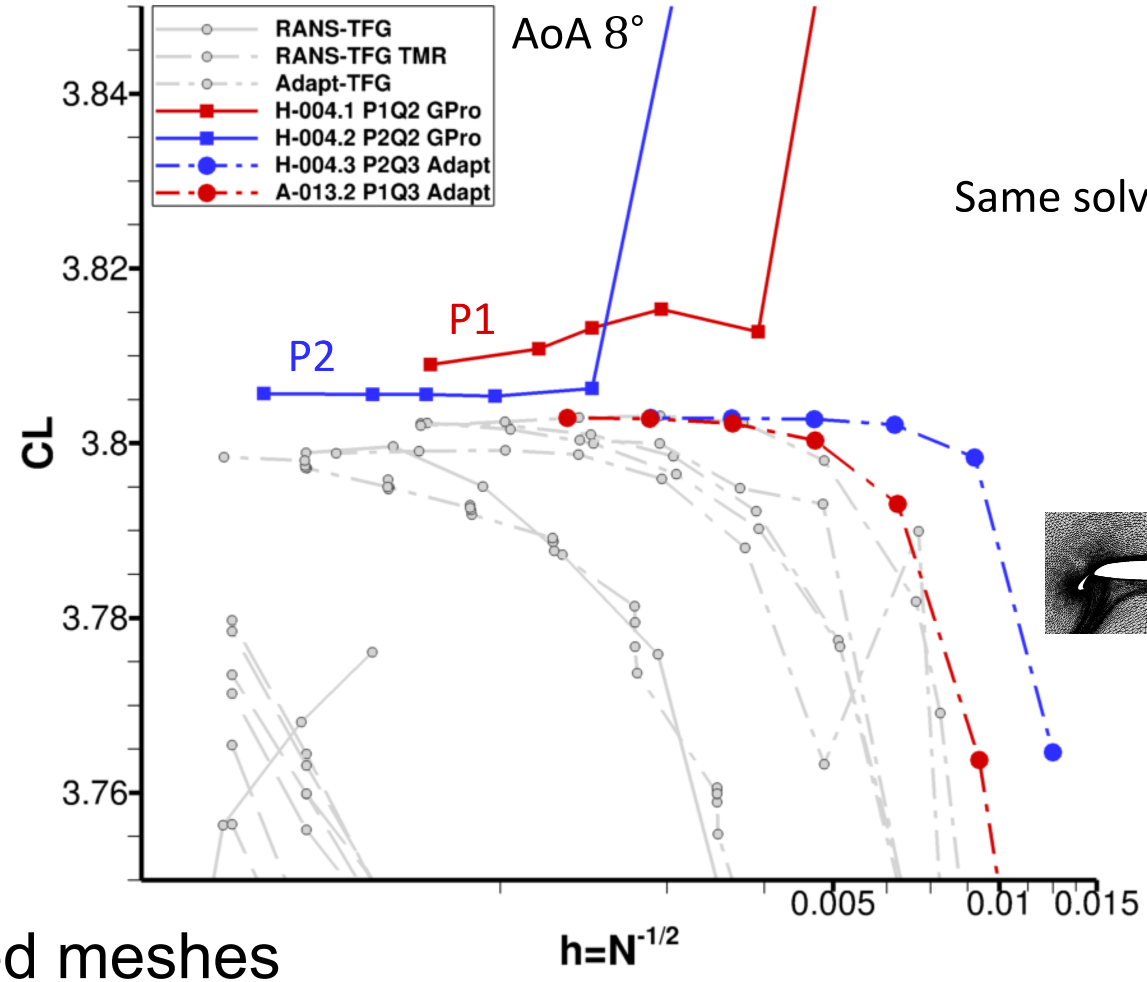
c_l error: 1.5×10^{-5}

Multi-Element High-Lift Airfoil Expert Meshes

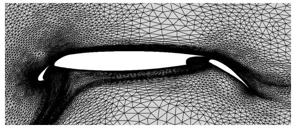
High-Lift 4 Results

Block structured





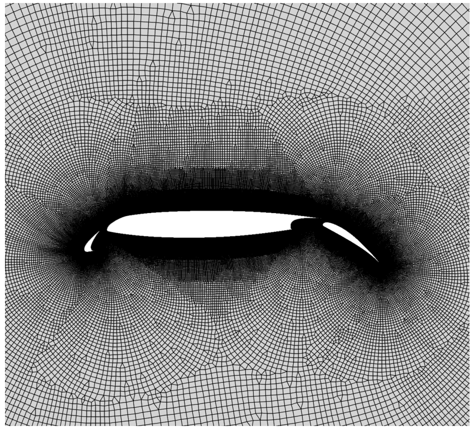
Same solver

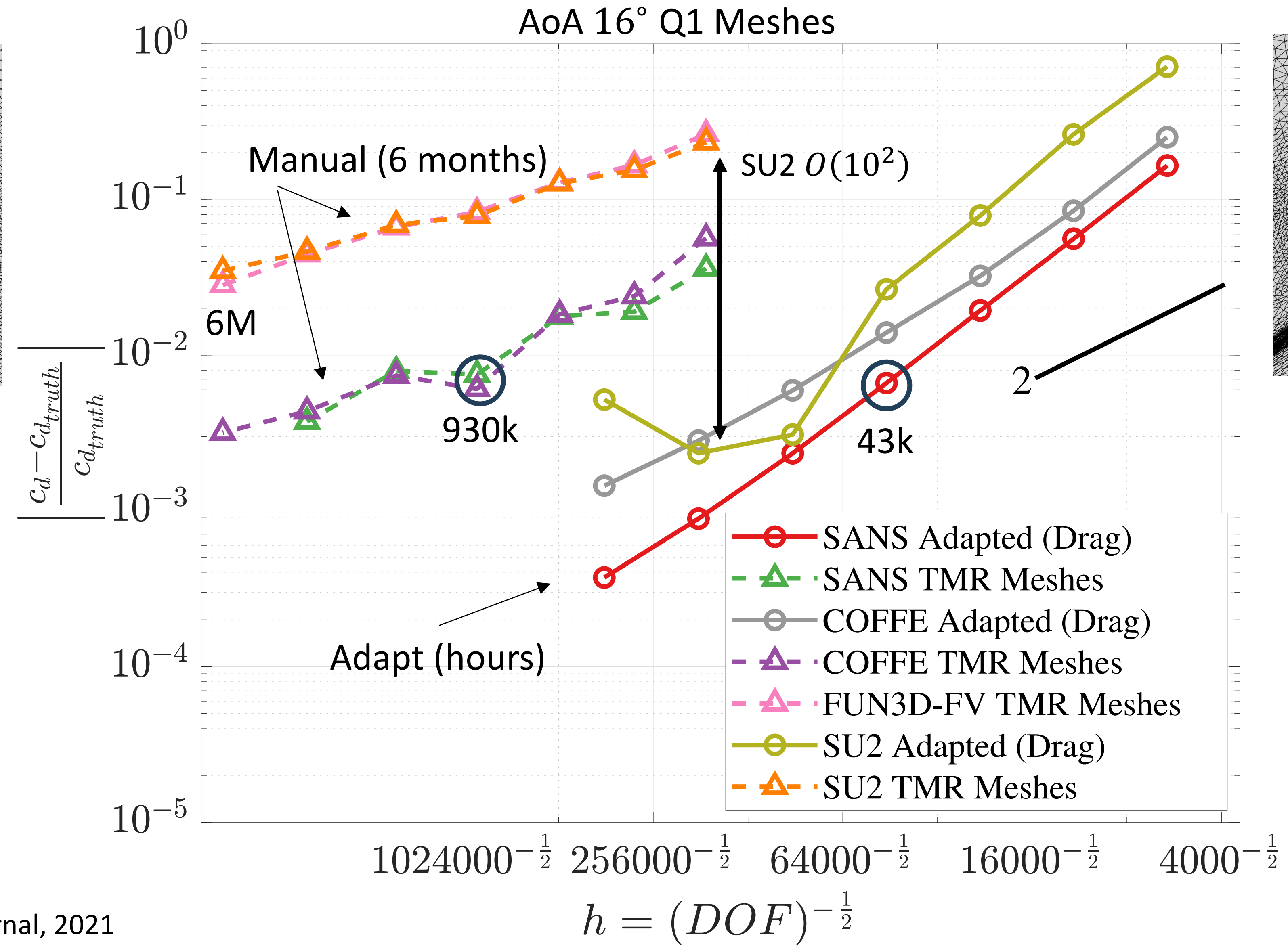


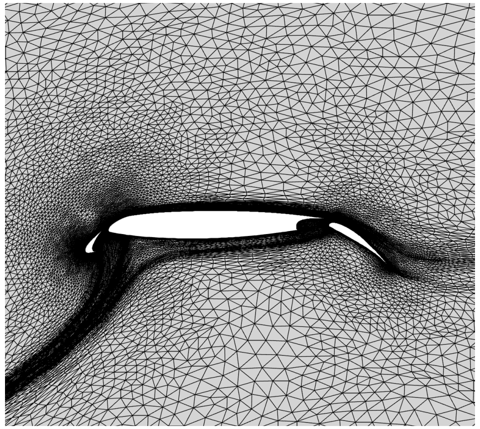
- P1 → P2: Reduced error on expert crafted meshes
- Expert and adapted meshes converge to different solutions?!
  - Expert mesh increases resolution where it does not improve lift
- Community lack of experience for manual curved mesh generation

Multi-Element High-Lift Airfoil Expert Meshes Cont.

Discretization: FV and FEM P1

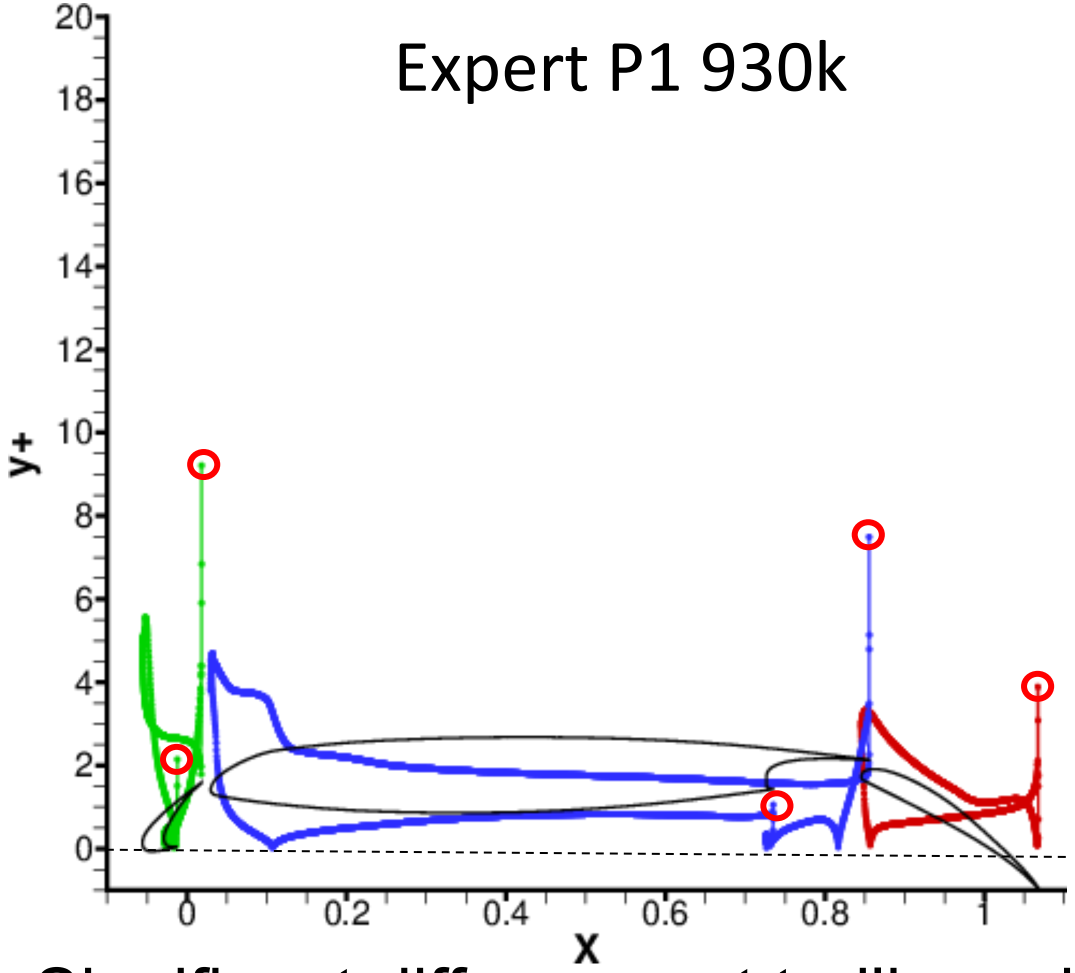


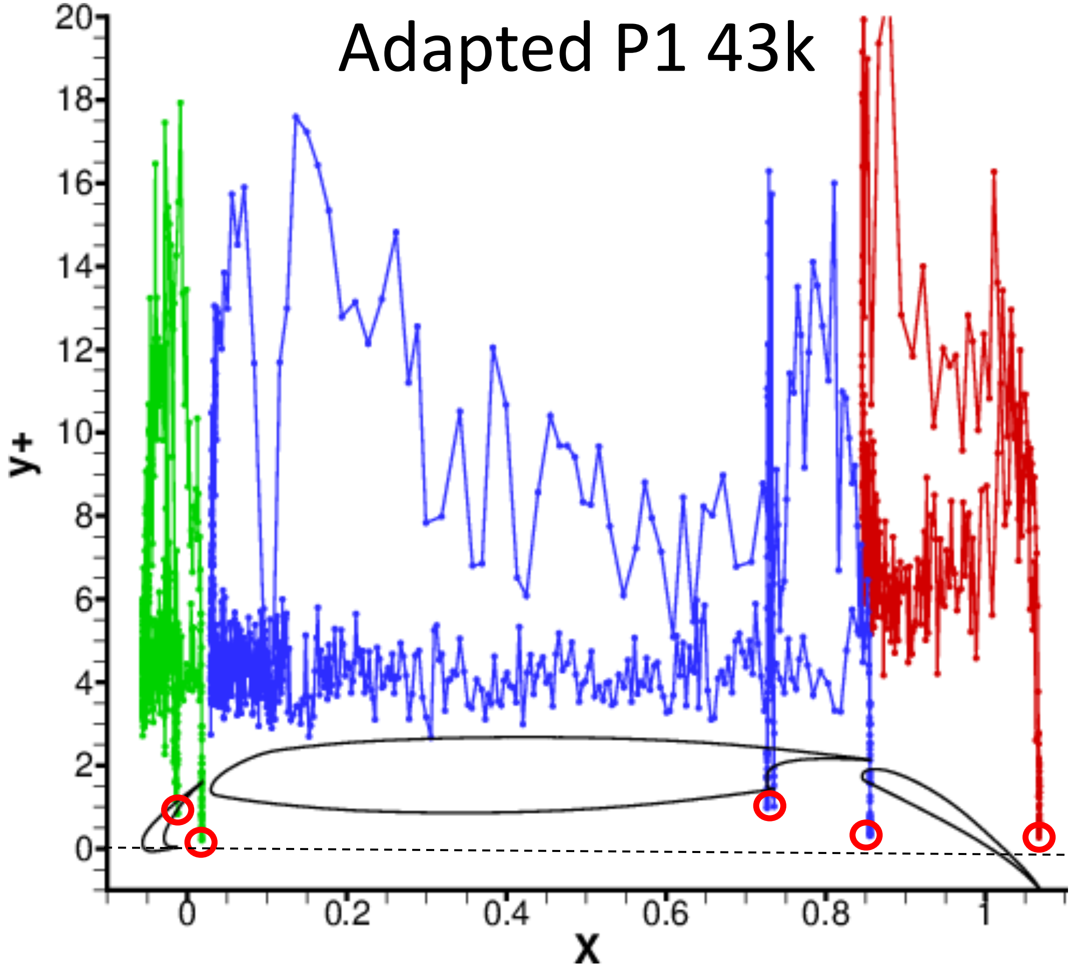




Galbraith et al., AIAA Journal, 2021

Expert and Adapted with Comparable Drag Error





- Significant difference at trailing edges
- $y^+ < 1$ is not correlated with accuracy for RANS-SA

High-Order TFG Participation

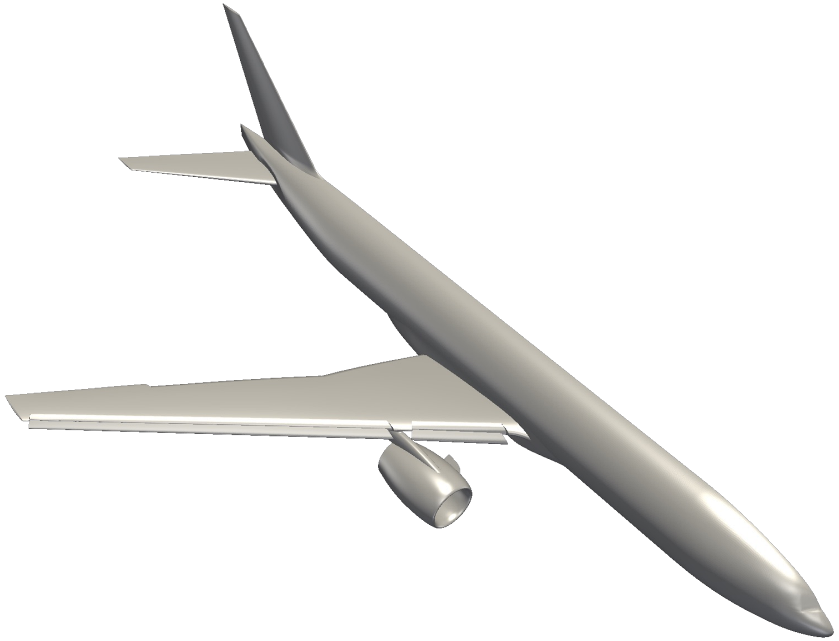
- 5-10 participants meeting every two weeks

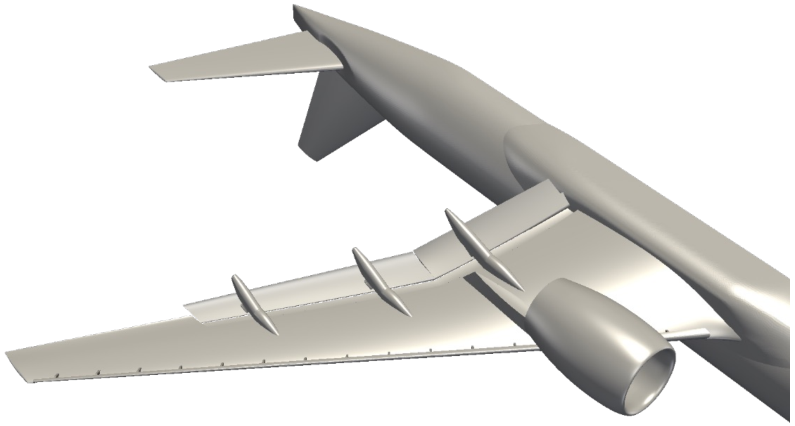
- 3 groups generating curved meshes
  - ANSA: WMLES and RANS Q2
  - Oak Ridge National Laborites, Pointwise: RANS Q2
  - Barcelona Super Computing Center (BSC): WMLES Q2-Q4

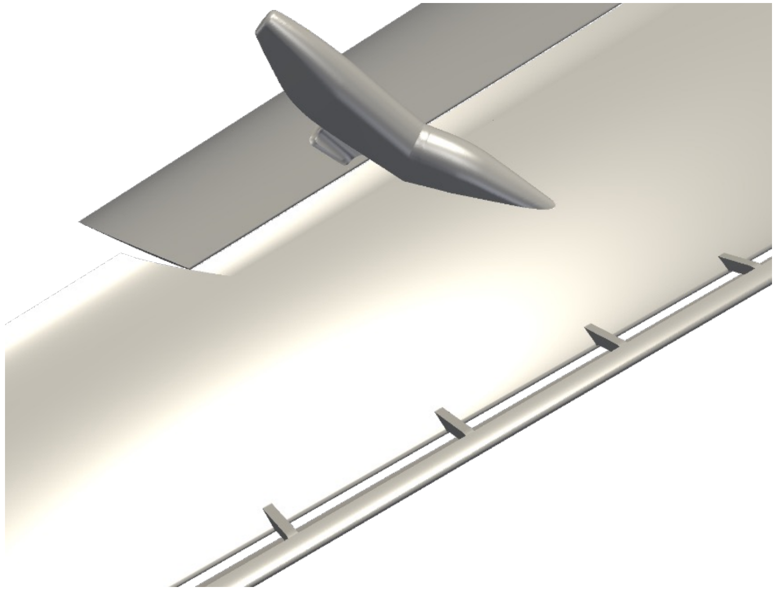
- 2 codes/groups submitted solutions
  - COFFE: University of Tennessee
  - hpMusic: University of Kansas

Challenges: 3D mesh curving for high-order methods

- **Human labor**: time to prepare a meshable model (everyone’s problem)
- **Efficiency**: linear mesh resolution limited by time & memory
- **Robustness**: geometric tangents & boundary layers lead to invalid curving
- **Flexibility**: handling complex geometries with wide range of length scales
- **Guarantees**: ensure extrema mesh quality and geometric accuracy



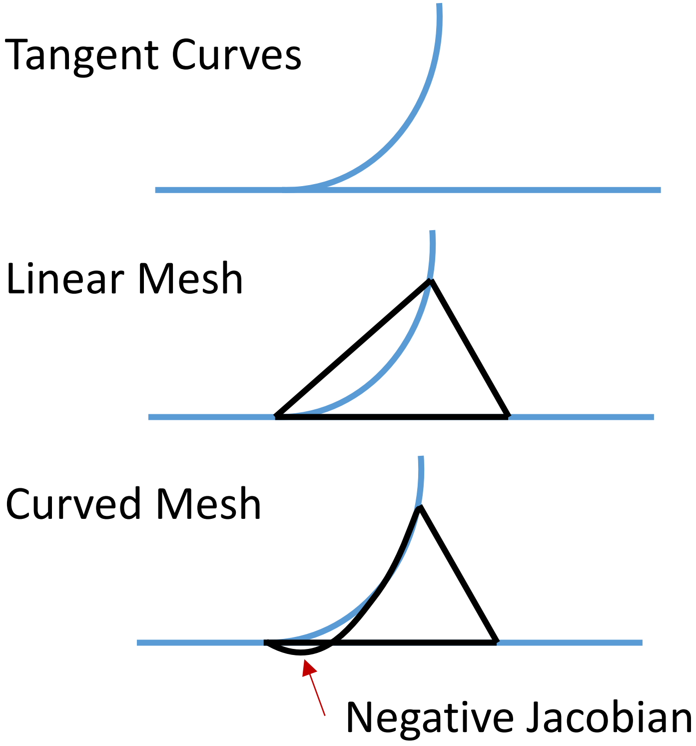


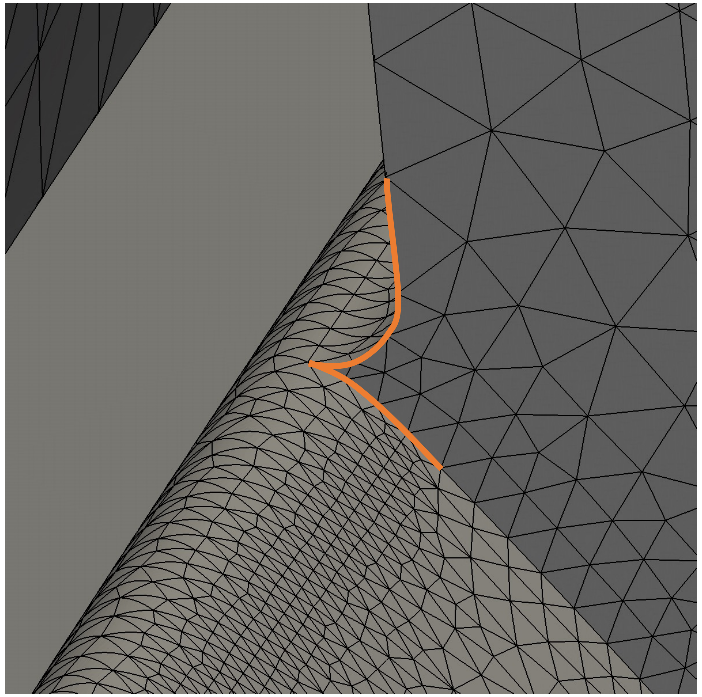


Challenges: Geometric Tangencies (Pinch Points)

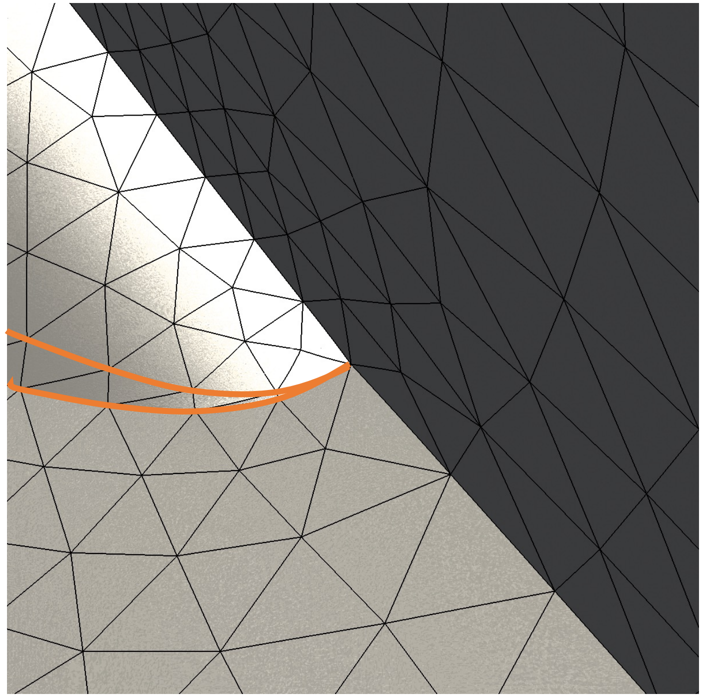
Unavoidable curve / surface tangencies hampers:

- **Convergence of curving solver:** More linear solver iterations & non-linear back-trackings
- **Mesh quality:** May lead to low-quality elements





Tangency on a bracket



Tangency on the vertical stabilizer

BSC Promises: 3D mesh curving for high-order methods

- **Human labor:** reduce preparation time using virtual geometry
- **Efficiency:** finer resolutions using a distributed curving solver
- **Robustness:** always an answer by ensuring intermediate validity
- **Flexibility:** handling complexity using curving for virtual geometry
- **Guarantees:** high-fidelity by optimizing quality & accuracy

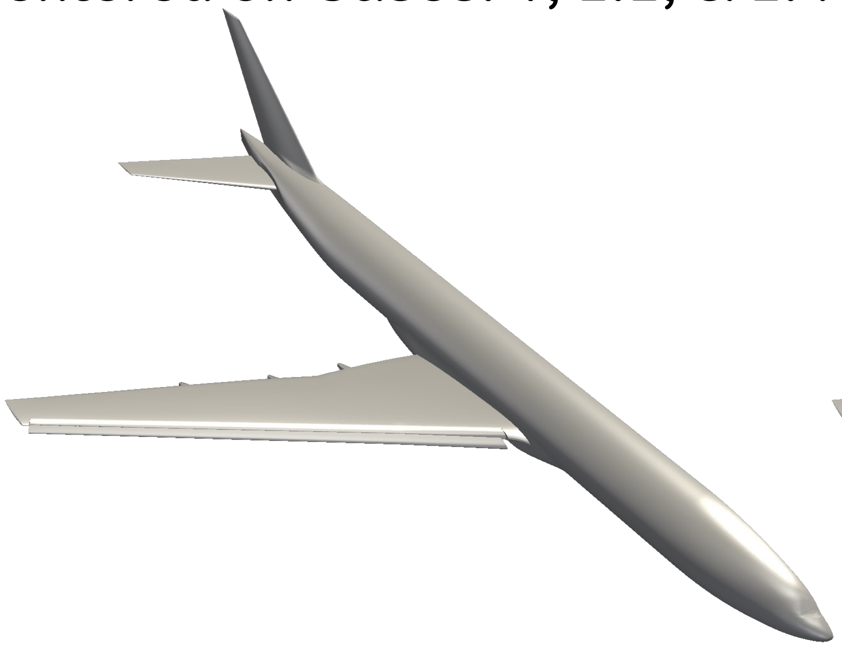
[Ruiz-Gironés, Roca AIAA’22; CAD’22]

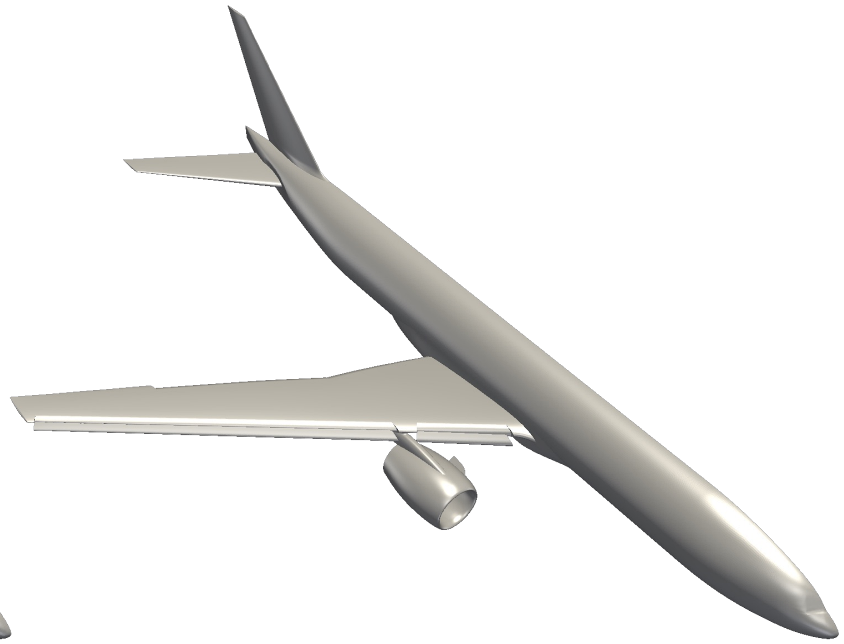
- **Value:** enabling high-fidelity aerodynamic simulations for unsteady implicit WMLES solvers

[Wang AIAA’24]

Cases 2.2 & 2.4: Geometry Definition

HO-TFG centered on Cases: 1, 2.2, & 2.4



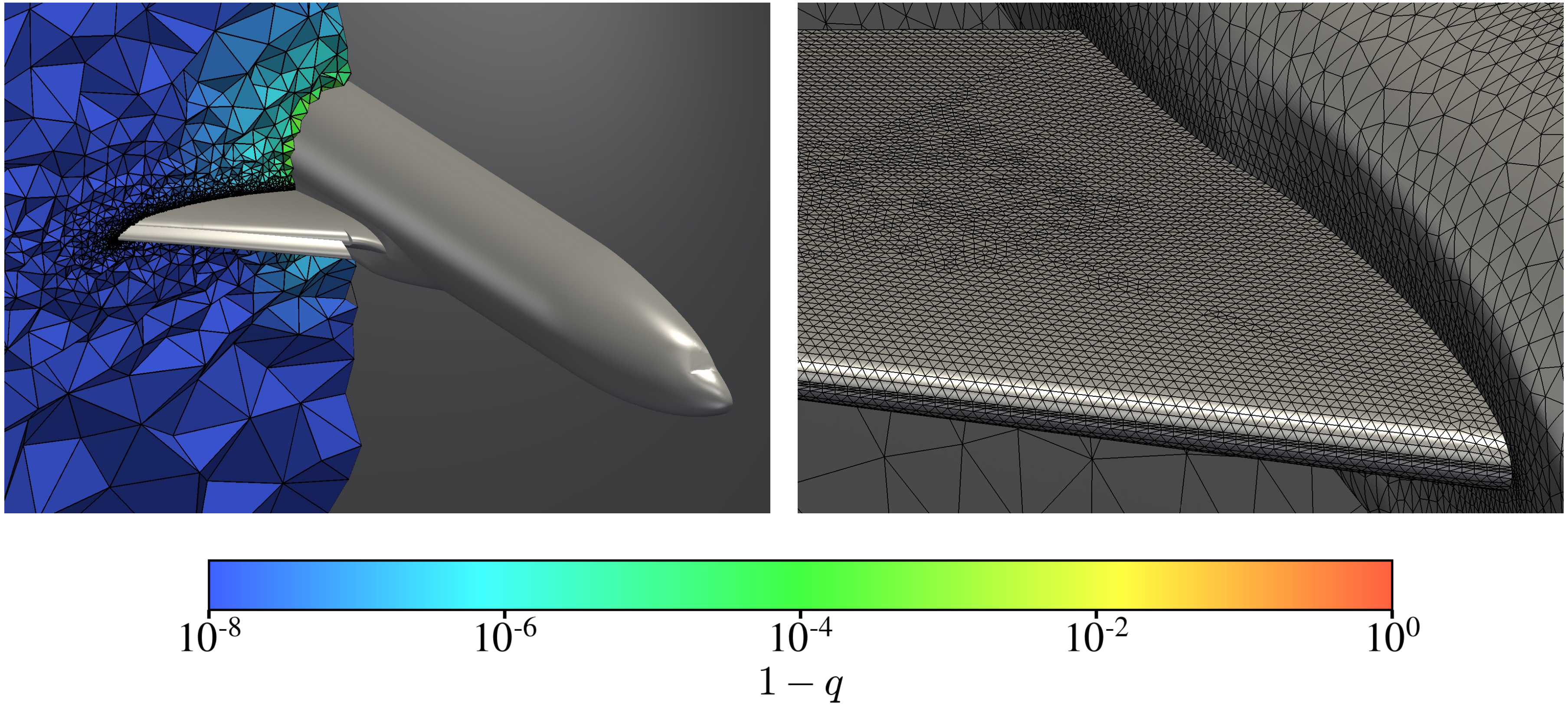


Case 2.2:
- Horizontal / vertical stabilizers
- Slats

Case 2.4:
- Horizontal / vertical stabilizers
- Slats
- Flaps
- Nacelle

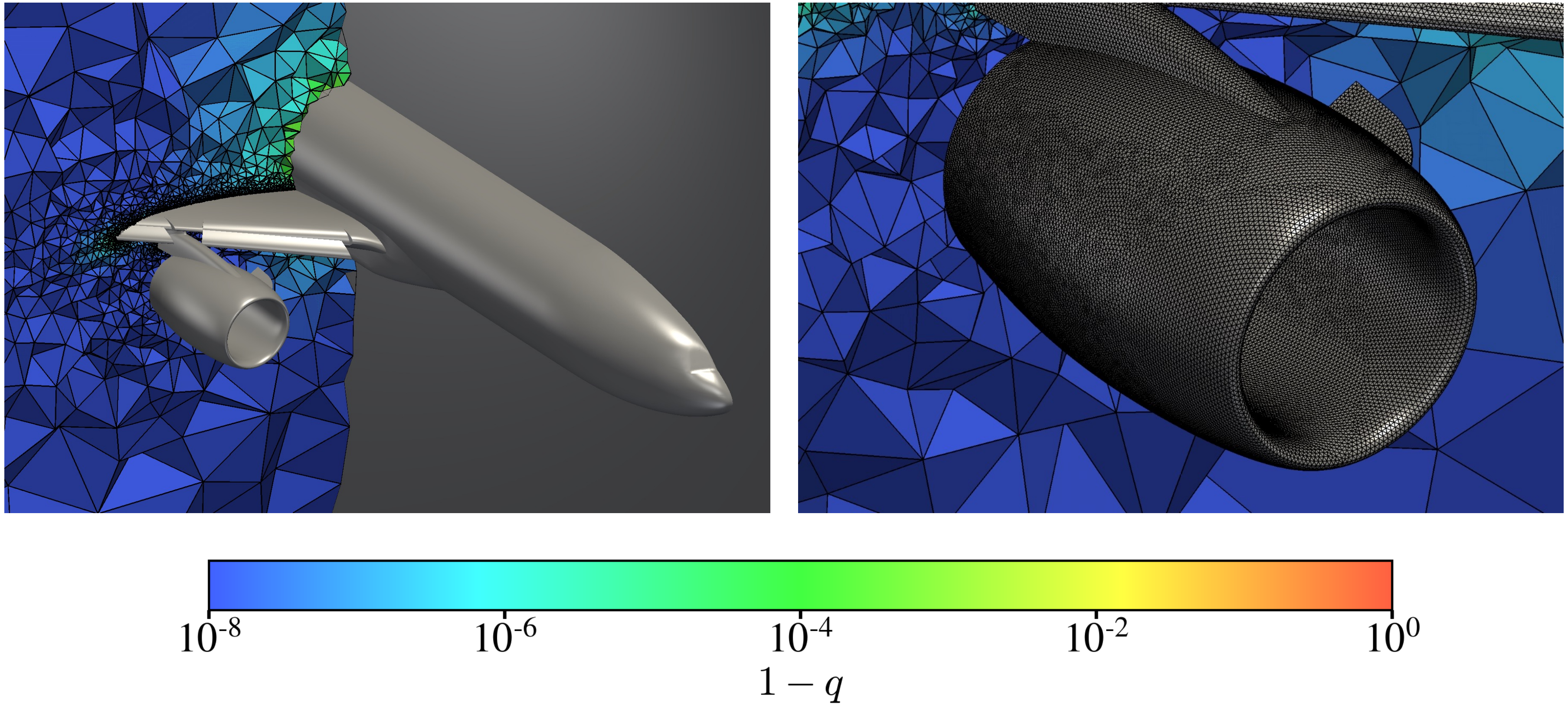
BSC Curved Mesh: Case 2.2

- **Mesh analysis**: Isotropic, Q2, 2.8M elements, 4.2M nodes
  - **Computational resources**: 30 minutes with 768 processors



BSC Curved Mesh: Case 2.4

- **Mesh analysis**: Isotropic, Q2, 3.6M elements, 5.4M nodes
  - **Computational resources**: 60 minutes with 768 processors

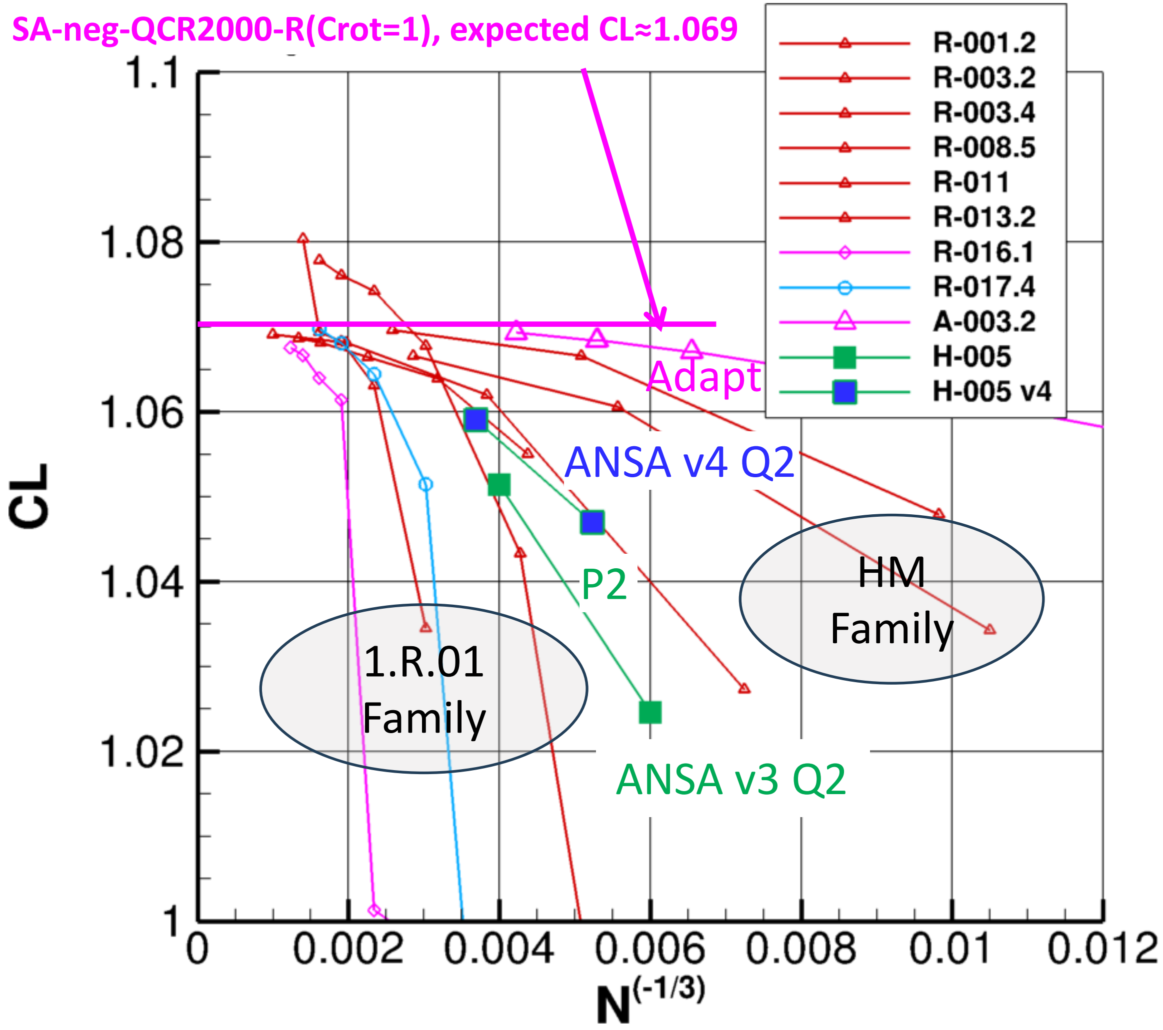


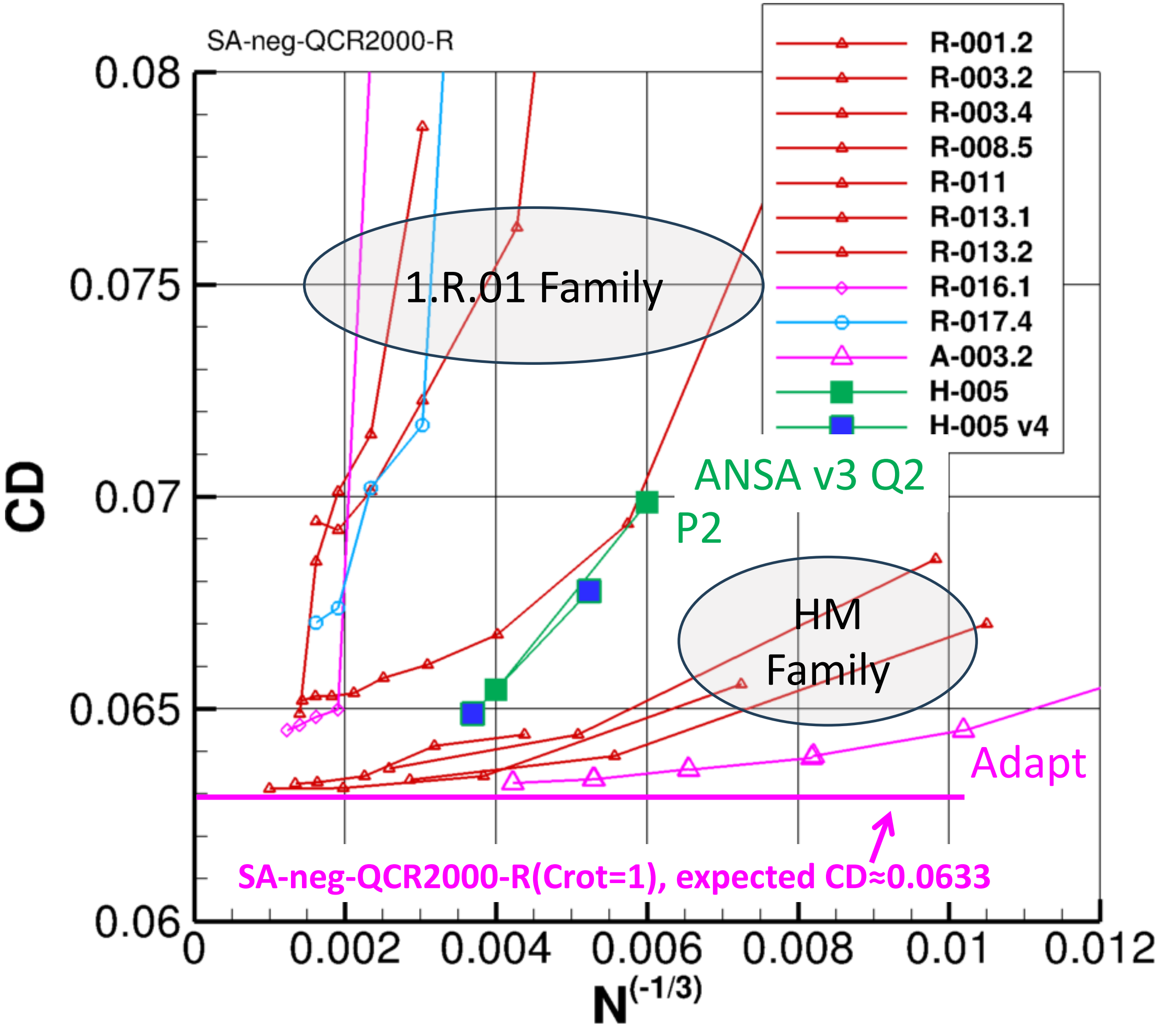
Submitted High-Order Solutions

- SUPG Finite Element Discretization
  - RANS P2 (3^{rd}-order)
  - Compressible meanflow & SA-neg-QCR2000-R (Crot=1) equations
  - Iso-parametric: P2Q2
  - Machine zero residuals

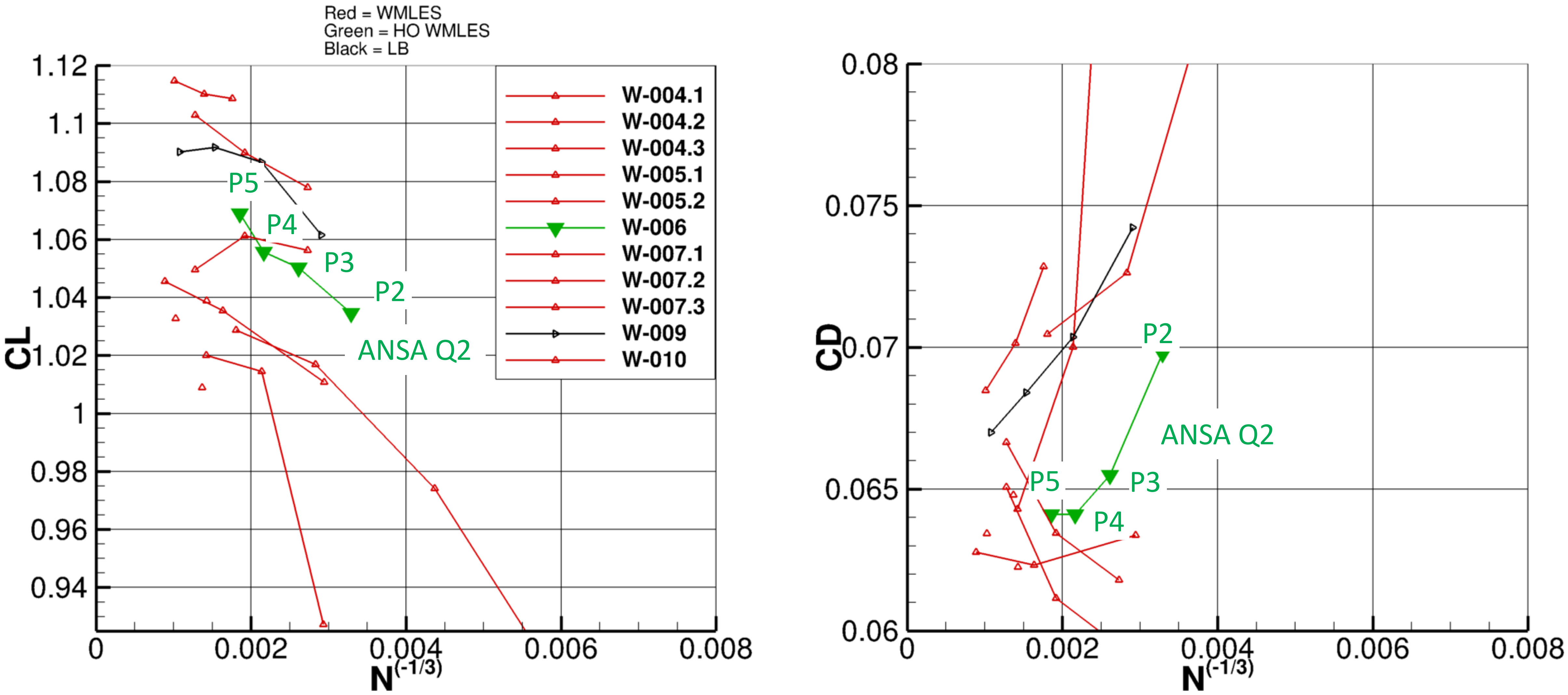
- Flux Reconstruction/CPR Discretization
  - WMLES: Equilibrium wall-model
  - Vreman SGS model
  - ANSA/BSC Q2 isotropic tetrahedral mesh

Case 1 RANS Solutions: CL and CD



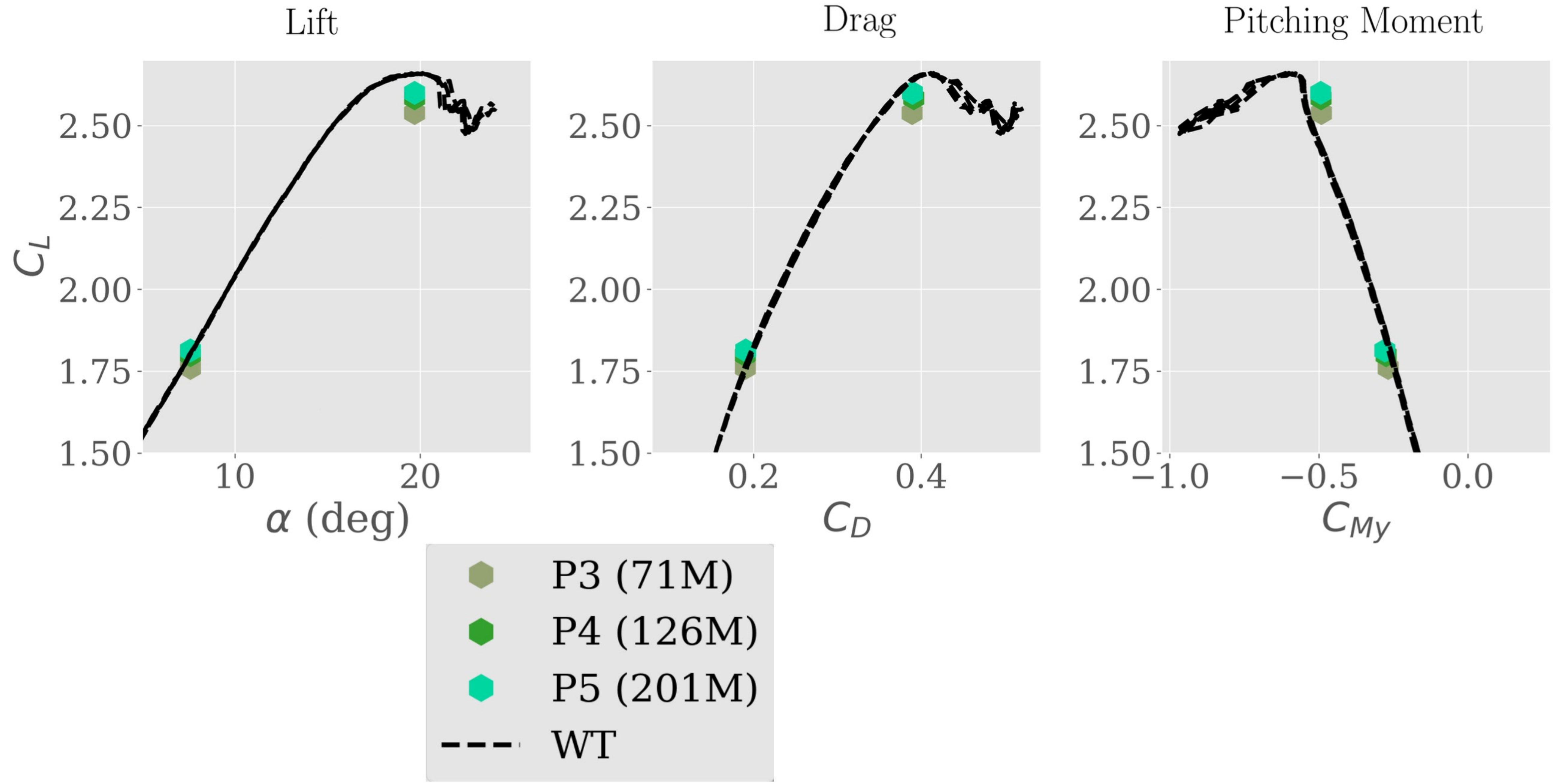


Case 1: WMLES CL and CD



- High-Order solution "in the middle of the pack"
- Grid convergence not observed by any WMLES solution

Case 2.4 BSC Q2 meshes: WMLES CL and CD



Summary

RANS

- High-order methods are silver bullet for meshing
  - The optimal Q1 mesh for P1 is not optimal Q2 mesh for P2
  - Fewer elements → distribution is more important

- High-order methods are effective in 2D with adaptation
  - Fixed meshes corrected based on interrogation of adapted meshes

- We need more information on “optimal” meshes in 3D
  - Adaptation can help inform
  - Curved adapted meshes is a research topic

WMLES

- High-order is promising
- Mesh convergence and mesh “optimality” not well understood

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- Spearheaded High-Order TFG and curved mesh generation

“You cannot solve what you don’t resolve!”

-- Steve Karman